DOCUMENT RESUME

ED 300 374 TM 012 076

AUTHOR Johns, Jennifer S.; Howden, Hilde
TITLE The APS Mathematics Program Curriculum Evaluation, 1985-86.
INSTITUTION Albuquerque Public Schools, NM. Planning, Research and Accountability.
PUB DATE Mar 87
NOTE 66p.
PUB TYPE Reports - Evaluative/Feasibility (142)

EDRS PRICE MF01/PC03 Plus Postage.
DESCRIPTORS Content Analysis; Counselor Attitudes; *Curriculum Evaluation; Educational Objectives; Elementary Secondary Education; Mathematics Achievement; *Mathematics Curriculum; Parent Attitudes; Principals; *Program Effectiveness; *School Districts; School Surveys; Student Attitudes; Teacher Response; Teaching Methods
IDENTIFIERS *Albuquerque Public Schools NM

ABSTRACT

The mathematics program of the Albuquerque (New Mexico) Public Schools (APS) was evaluated during the 1985-86 school year. Data were collected via surveys, interviews, analyses of student achievement data, and reviews of publications. The subjects were 506 teachers, 90 counselors, 1,502 students, 100 principals, and 965 parents. Evaluation questions focused on goals, content, teaching strategies/materials, and effectiveness. Teaching basic math knowledge, thinking, and problem solving; preparing students for further education and jobs; and enabling students to use math in daily life were all considered to be very important by the subjects. Also important was encouraging students to enjoy math for its own sake or in their leisure time. Specific concerns were: (1) curriculum modification; (2) needs of special groups of students; (3) more mathematics for those not bound for college; (4) incorporation of consumer mathematics into preparation of college-bound students; (5) excellence of textbooks; and (6) time spent in instruction at the elementary level. School personnel and parents rated the program as somewhat effective in all of its goals. Student achievement results showed above average performance with steady growth, although some algebra scores were below the national average. Students were fairly positive and confident about their abilities, with elementary students the most and high school students the least positive. Nine data tables and six graphs are provided. (SLD)

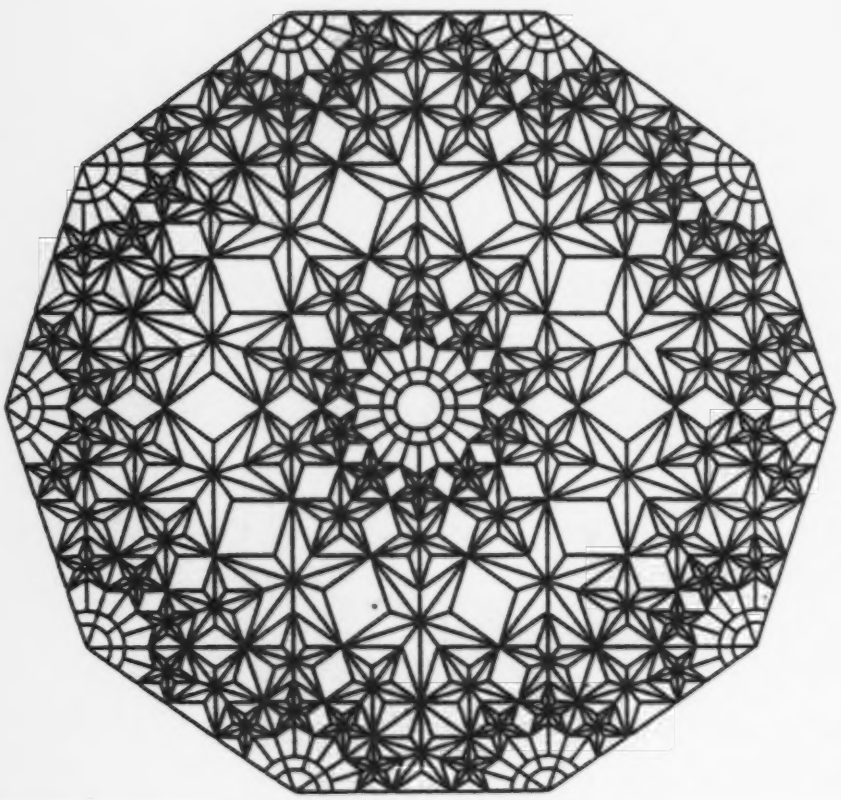
****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED300374

The APS Mathematics Program

Curriculum Evaluation

1985–86



U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

☑ This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

C. ROBINSON

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

APS
ALBUQUERQUE PUBLIC SCHOOLS

TM012076

PLANNING, RESEARCH AND ACCOUNTABILITY

ALBUQUERQUE PUBLIC SCHOOLS

725 University, S.E.
Albuquerque, New Mexico 87106

BOARD OF EDUCATION

IRA ROBINSON
President

LENORE WOLFE
Vice-President

MARY LEE MARTIN
Secretary

DIEGO GALLEGOS
Co-Chairman, Finance Committee

ED MARINSEK
Co-Chairman, Finance Committee

LEONARD DE LAYO, JR.
Member

PAULINE MARTINEZ
Member

LILLIAN C. BARNA
Superintendent

MARY K. NEBGEN
Deputy Superintendent, Instructional Services

PLANNING, RESEARCH, AND ACCOUNTABILITY
Carol Robinson, Director
Sandra O'Neal, Assistant Director for Testing and Evaluation

Prepared by:

Jennifer S. Johns
Program Evaluation Specialist
with
Hilde Howden
District Math Coordinator

Special thanks to Lois Folsom, Gary Mayhew, Gene Jercinovic, and Dorothy Sadousky for their cooperation and valuable assistance.

* * *

Thanks also to the elementary math representatives, the middle school content leaders, the high school department heads, and all those who responded to the surveys for their contributions of time and expertise.

MARCH, 1987

The APS Mathematics Program
Curriculum Evaluation
1985-86

TABLE OF CONTENTS

Administrative Summary..... 1

Evaluation Design..... 4
Table 1 Sampling Plan for Math Evaluation..... 7
Table 2 Response Rates on Math Evaluation..... 10

History of Math Education..... 14

Goals..... 22
Table 3 Recommended Involvement of Various Groups in Determining APS Math Goals..... 22
Table 4 Rankings of Importance of Mathematics Goals..... 23

Content..... 24
Table 5 Recommended Involvement in Determining Math Content..... 25
Table 6 Appropriateness of Calculus for High School Students..... 32

Teaching Strategies..... 34

Math Materials..... 37
Table 7 Percent of Teachers Reporting Daily and Weekly Use of Math Materials..... 37

Teacher Preparation..... 44

Effectiveness..... 46
Figure 1 Third Grade CTBS Scores, 1983-86..... 48
Figure 2 Fifth Grade CTBS Scores, 1982-86..... 49
Figure 3 Eighth Grade CTBS Scores, 1982-86..... 49
Figure 4 Third Grade Key Competencies, 1984-86..... 51
Figure 5 Fifth Grade Key Competencies, 1984-86..... 51
Figure 6 Eighth Grade Key Competencies, 1984-86..... 52
Table 8 Scott, Foresman Algebra Test Results Summary, 1985-86..... 53
Table 9 Scott, Foresman Algebra Test Results in Percentile Scores, 1985-86..... 54

Summary and Conclusions..... 55

References..... 59

Administrative Summary

The APS mathematics program was evaluated during the 1985-86 school year in accordance with the curriculum review cycle. The evaluation design utilized surveys, interviews, analyses of student achievement data, and reviews of publications on mathematics education to identify the areas in which the APS math program is effective and the areas in which modifications are needed. Data were collected from teachers, students, principals, and parents at the elementary, middle, and high school levels; from secondary counselors; and from community employers.

The response rates to the evaluation instruments were quite high. Seventy-six percent of teachers, 75% of counselors, 65% of principals, 89% of students, and 56% of parents who were selected, responded to the surveys. Seven of the 11 businesses selected participated in interviews. All of the groups surveyed believed that those individuals who are closest to the classroom and who have the most specialized knowledge of mathematics, that is classroom teachers and district math specialists, should be the most involved in determining the goals and content of the APS math program, as well as in judging its effectiveness. Survey respondents rated the following goals as very important: teaching basic mathematical knowledge, thinking, and problem solving; preparing students for further education and for jobs; and enabling students to use math in their daily lives. Less important, but still thought to be somewhat important, were encouraging students to enjoy math for its own sake or in their leisure time.

All of the groups surveyed reported moderate familiarity with the content of the math program at all levels although elementary teachers were less familiar with the secondary math program than were secondary teachers with the elementary math program. In spite of parents' reported familiarity with the math program, a third of the parents felt communication between the school and the home about math was inadequate. Parents at all levels desired information on math content, goals, and expectations so they would have an idea of what their children should master in a year. In addition, while recognizing their responsibility for initiating or improving communication, parents wanted more contact with mathematics teachers in person or in writing. Parents were especially concerned about early notification if their children were experiencing problems in mathematics.

A few areas of the math curriculum need additional emphasis or modification according to school personnel, parents, and professional math organizations. First, the math program at all levels needs to emphasize problem solving skills, the application of mathematics to daily life, mental calculations and estimation, and compilation and interpretation of data. Second, the needs of certain groups of students must be addressed. These are developmental/ remedial students, low achievers, ESL students, and students who are not in the gifted program but need enriched mathematics activities. Third, the high school program needs to be modified in order to expose non-college bound students to a broader range of mathematics and to incorporate consumer mathematics into the preparation of college-bound students. The role of calculus in the high school math program needs further clarification.

Elementary math instruction in APS is characterized by a greater diversity of teaching strategies and materials than is secondary math instruction. The typical elementary math class, while relying heavily on textbooks, also frequently incorporates manipulatives, worksheets, and computers. Textbook assignments and drill and practice are the most frequently used strategies, but elementary teachers also use discussion, chalkboard work, small group work, peer tutoring, homework, and individual student-teacher conferences. At the secondary level, the textbook is the dominant instructional material, although other math books, worksheets, and calculators are frequently used. Secondary teachers rely primarily on textbook assignments, homework, lecture, chalkboard work, drill and practice, and discussion, with occasional use of peer tutoring and individual conferences.

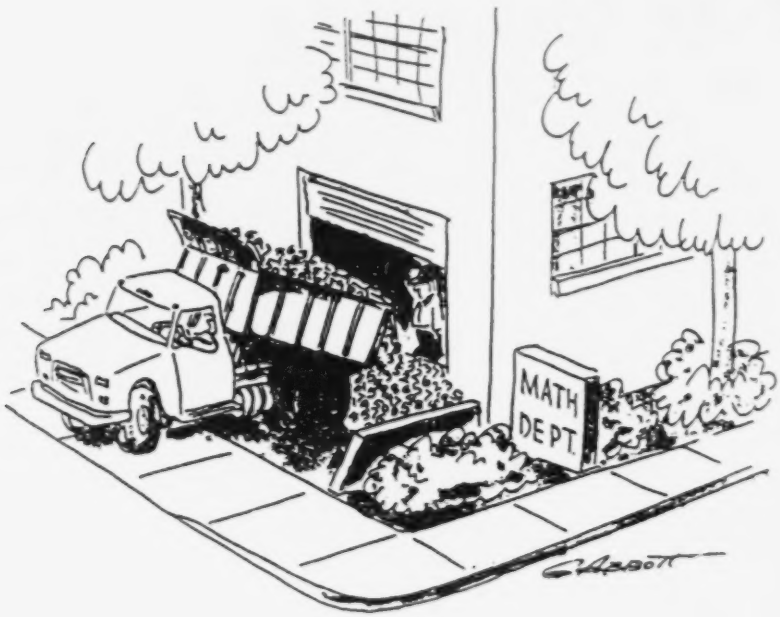
Since the textbook is relied upon so heavily by classroom teachers, it is important that excellent textbooks are selected. According to survey results and national recommendations for mathematics, math textbooks should emphasize higher order mathematical skills such as problem solving and applications in addition to computational skills and basic mathematical knowledge. Teachers need to become better informed regarding materials selection procedures, including textbooks. In addition, an attempt should be made to incorporate more diverse materials into the mathematics classroom by providing teachers with information on the applications of manipulatives, calculators, and computers. Lack of access to these materials may partially explain their limited use; the acquisition of more manipulatives and calculators may be necessary.

Another area that warrants attention is the amount of time spent on mathematics instruction at the elementary level. A great deal of variation existed, from 15 to 60 minutes a day, with an average of 46 minutes a day spent on mathematics. Since Senate Bill 106 mandates an hour each day of mathematics instruction in grades one through three, and five hours weekly in grades four and five, instructional time may need to be modified. It would be helpful to clarify for teachers how to figure instructional time. Teachers may not be including in their calculations the time spent by students on mathematics activities in other content areas.

School personnel and parents rated the APS math program as somewhat effective in achieving all of its goals. Student achievement results indicate that APS elementary and middle school students scored above the national averages on standardized tests in mathematics computation and applications. For the most part, these students have shown steady growth over the last four years. Proportionately more APS students master whole numbers, geometry, charts and graphs, fractions, and decimals than master measurement and number theory on the Key Competencies. The high school students who participated in optional algebra testing scored slightly below the national average, with an average percentile score of 46 in 1986.

Overall, students are fairly positive about mathematics and feel confident of their ability. They believe mathematics is useful outside of school and many of them think they would enjoy using math in their careers or vocations. Elementary students were the most positive, and indicated they enjoyed tackling difficult problems and experimenting with mathematics. However, by middle school, student attitudes changed dramatically. Middle school students were less confident, less interested, and had lost the enthusiasm for problem solving and experimention shown by elementary students. The attitudes declined again, though only slightly, between middle school and high school.

While some of this change in attitude may be explained by changes in the students themselves, it is reasonable that at least part of this change in attitude is due to students' experiences in school. This is an area that requires more investigation. Perhaps altering instructional approaches or adding variety in materials would make mathematics more rewarding, challenging, and enjoyable for students.



Cartoons from Phi Delta Kappan

Evaluation Design

Role of Evaluation in the Curriculum Cycle

In 1981, the Albuquerque Public Schools (APS) adopted a curriculum planning process for all curricular areas. Each curricular area, such as mathematics, is rotated through a six-year cycle which involves adoption of new materials, implementation of the revised program, evaluation, and pre-adoption activities. The process is important because it allows for cyclical, systematic revision of curriculum throughout the cycle. The opinions of many groups of people, typically teachers, principals, students, and parents are sought, thereby improving the quality of the decisions that are made. The evaluation component of this cycle precedes the adoption of new materials to ensure that this information is available to help revise the curriculum guides and aid in the selection of new materials.

A curriculum evaluation is not an evaluation of teacher effectiveness, but rather an evaluation of how well the curriculum currently is meeting the needs of students and teachers. The evaluation provides an opportunity to identify areas of strength as well as areas of need. Four major topics are addressed in curriculum evaluations: goals, content, teaching strategies and materials, and effectiveness. By focusing on each of these areas, a comprehensive evaluation is possible.

Process for Developing and Implementing the Evaluation

To ensure that the math evaluation addressed topics of importance in mathematics, a variety of strategies were used. First, major studies of mathematics by national organizations such as the National Council for Teachers of Mathematics (NCTM), the National Science Foundation (NSF), and the National Assessment of Educational Progress (NAEP) were reviewed. Second, the curriculum coordinator for mathematics and the district itinerant math teachers were queried. Third, meetings were conducted in November of 1985 with all of the APS math representatives for elementary, middle, and high school. At these meetings, teachers generated questions in each of the four major evaluation areas and suggested from whom information should be collected and how the evaluation might be structured to ensure success. All of this information was analyzed, resulting in a set of questions which provided the framework for the evaluation. This set of questions was then reviewed in draft during February of 1986 by a group of principals, a teachers' advisory council, and the Albuquerque Public Schools/University of New Mexico clinical supervisors. Finally, revisions were made based on the suggestions of these groups.

Following this, the sampling plan was developed. It was decided, based on teachers' recommendations, that evaluation information would be collected from teachers, counselors, principals, parents, students, and business representatives. The sampling plan identified exactly from whom information would be collected, how it would be collected, and when it would be collected.

In March of 1986, the math representatives received information on this process, which they communicated to their schools. During April, questionnaires were developed and reviewed in draft by appropriate groups,

such as teachers and parents. Revisions were made and surveys were printed and given to the math representatives at meetings, for distribution in their schools. In April and May, the surveys were completed and returned by the math representatives to the Planning, Research, and Accountability (PRA) department, where they were sorted and submitted to Data Services for keypunching.

Interviews with business representatives were conducted by three APS itinerant teachers during the months of June, July, and August. In the fall, results of surveys and interviews were discussed with the curriculum coordinator and itinerant teachers to note important trends and to provide for immediate feedback. In September, additional data analyses were performed and work was begun on the evaluation report. Plans were made to review the report in draft with the coordinator and itinerant teachers and to disseminate the results of the evaluation to various groups.

Evaluation Questions

The questions which provided the framework for the evaluation were grouped in the four evaluation areas: goals, content, teaching strategies and materials, and effectiveness. The questions are listed below.

Goals

- What are the goals of the APS mathematics program? Do they provide adequate direction?
- How do the APS goals compare to goals recommended nationally?
- How familiar are teachers, administrators, and parents with the goals of the APS math program? How much concurrence is there among these groups about recommended goals for mathematics education?
- How should the goals for the APS math program be determined?

Content

- How is the content of the APS mathematics program determined? How should it be determined?
- Is the scope and sequence of the APS mathematics curriculum appropriate?
- Is the math curriculum flexible and diverse enough to meet the needs of all students?
- Does the APS mathematics program emphasize content and skills deemed important by students, parents, teachers, administrators, and mathematics specialists?

Teaching Strategies and Materials

- What strategies and materials are being used in math classes? How frequently are various strategies and materials used?
- What is the process for selection and purchase of materials? How effective is the process? How familiar are teachers and administrators with the process?
- Are materials adequate in terms of quality, quantity, and appropriateness for various grade levels and student needs?
- What procedures are used to place students in mathematics classes? How adequate are they?
- What is the background and experience of APS mathematics teachers?
- What are the most effective ways to provide teachers with information on new content, strategies, and materials?

Effectiveness

- How is student performance evaluated in mathematics? Is this adequate?
- How should the effectiveness of the APS mathematics program be determined?
- How effective is the APS mathematics program? How effective is it perceived to be by teachers, students, parents, administrators, and the community?
- How adequately does the APS math program prepare students for their future schooling, careers, and leisure time?

Instruments and Sampling Procedures

Evaluation data were collected from six groups: teachers, principals, counselors, students, parents, and representatives from several of Albuquerque's largest employers. Questionnaires were used to collect data for all school personnel, students, and parents because they are advantageous in two major ways. First, they provide information from a larger number of people than is possible with other methods, due to the lesser amount of time required for data collection and analysis. Second, they provide an opportunity for people to express their true opinions because they maintain anonymity. The business representatives were interviewed individually to facilitate receiving information from this group and to provide an opportunity for exchange between APS teachers and the business community. A description of the groups sampled and the number of individuals in each group is presented in Table 1 on the following page.

Table 1
Sampling Plan for Math Evaluation

	Elementary	Middle	High
Teachers	All teachers at 18 elementaries N = 347	Math teachers at all middle schools N = 145	Math teachers at all high schools N = 156
Counselors	None	Counselors at all middle schools N = 47	Counselors at all all high schools N = 72
Principals	All principals N = 74	All principals N = 49	All principals N = 26
Students	One randomly selected 4th grade class from 17 elementaries N = 394	Two randomly selected math classes N = 582	Four randomly selected math classes N = 752
Parents	Parents of one randomly selected 4th grade class from 18 elementaries N = 441	Parents of students in classes above N = 571	Parents of students in classes above N = 730

Business Representatives	*BDM Corporation *City of Albuquerque General Electric Aircraft Engine Group *Mountain Bell Public Service Company of New Mexico Sandia National Laboratories Southwest Community Health Services Sperry Corporation Sunwest Bank of Albuquerque *United States Postal Service University of New Mexico N = 11

*Unable to complete interviews

Since three curriculum evaluations are occurring simultaneously in any given year, PRA carefully coordinated the data collection to reduce duplication of sites or individuals. In addition, in response to the recommendations of the math representatives, the questionnaire items were predominately multiple choice, but with space provided for comments. This approach decreased the amount of time required to complete the survey, but allowed those who wished to make comments the opportunity to do so.

Teachers. Typically, the elementary school teacher is the most impacted by curriculum evaluations, since she or he teaches all subjects. Therefore, the elementary schools were divided so that some were involved in the music evaluation and the remainder were involved in the math evaluation. The 53 elementary schools available for the math evaluation were further divided into three groups of schools which were similar on three demographic factors: the percent of children receiving free lunch, the school stability rates, and the student scores on the math portion of the Comprehensive Tests of Basic Skills (CTBS). Each group completed either a teacher, student, or parent evaluation so that all elementary schools were involved in the evaluation, but only in one area. Teachers at 18 elementary schools were requested to complete the teacher survey which contained 40 multiple choice items. At the middle and high school level, all math teachers were asked to complete surveys since this group was rather small and did not overlap with other curriculum evaluations. The secondary teacher survey was identical to the elementary teacher survey with the addition of 8 items relevant to secondary mathematics.

Counselors. All counselors at the middle and high school level also were requested to complete surveys because of their involvement in student placement, career counseling, and college preparation. Elementary counselors were not included in the evaluation design. The counselor survey was based on the secondary teacher survey and contained 32 multiple choice items and one open-ended question.

Principals. Principals at all schools and assistant principals at the middle and high school levels were surveyed because of their role as the instructional leader. The principal survey contained a core of items from the teacher survey as well as three open-ended questions relevant to principals.

Students. Students were surveyed in the following manner. At the elementary level, a randomly selected class of fourth graders was surveyed at 17 elementary schools. Random selection by classroom makes the administration of surveys much easier for teachers since the entire class is involved in the activity, rather than a few students from several teachers' classes. Fourth grade was chosen because first and second graders are too young to complete written surveys and third and fifth graders are involved in CTBS testing every spring. The elementary student survey was primarily an attitude survey designed to ascertain students' attitudes toward the field of mathematics and toward math teaching strategies and materials. In addition, some demographic information was collected on the students' favorite subject as well as career aspirations. The student survey consisted of 32 multiple choice items.

At the middle school level, two groups of seventh grade students completed surveys: students in regular math clases and students in enriched math classes. One class of each was selected from all schools. Seventh grade was chosen because these students were not being tested with the CTBS and because

research indicates it is at this age that math attitudes often begin to change. It was believed that this approach would result in a good cross-section of students with varying levels of math skills. The middle school student survey was identical to the elementary survey with the addition of two items about homework.

At the high school level, four types of math classes were surveyed: basic, algebra I, geometry, and algebra II honors. Three randomly selected classrooms of students at each high school participated in the evaluation. Geometry and algebra I classes were surveyed at all high schools while basic and algebra II honors classes were each surveyed at half of the high schools. Again, it was believed that this approach would tap the opinions of many different kinds of students and therefore, be representative of students as a whole. The high school student survey consisted of 41 multiple choice items.

Parents. Parents of elementary, middle, and high school students were involved in the math evaluation. At the elementary level, the parents of 18 randomly selected classes of fourth graders were asked to complete a survey which was hand-carried by the students. At the middle school level, parents of those seventh graders who completed student surveys, were asked to respond to a questionnaire. At the high school level, the same approach was used with parents of students in basic, algebra I, geometry, and algebra II honors classes. The survey consisted of 29 items, many of which were identical to items on the surveys of school personnel. In addition, parents were asked about their math backgrounds, about communication between the home and the school, and about how comfortable they felt helping their child with math homework.

Employers. A list of the 20 largest employers in the Albuquerque metropolitan area was obtained from the Chamber of Commerce. Eleven of these businesses were selected because of the likelihood of their needing APS graduates with mathematical skills. Each business was telephoned to identify the appropriate person for the interview, usually a manager in the personnel department. Interviews were conducted individually at the work site. Employers were asked if they hired APS graduates immediately after high school and, if so, how qualified they felt these individuals were. They were also asked how they evaluated students' educational backgrounds in their hiring process. The employers were queried on their knowledge of the APS mathematics program and their involvement with the public schools, as well as the type of employee mathematical skills they anticipated needing in the future. Finally, employers were provided an opportunity to ask any questions about the mathematics program in APS.

Response Rates. In spite of the length of the surveys, the overall response to the math questionnaires was quite good. Seventy-six percent of the elementary teachers responded, while 80% of the middle and high school math teachers completed questionnaires.

Seventy-five percent of counselors and 67% percent of principals and assistant principals completed surveys. At the elementary level, 96% of the selected students completed a questionnaire, while at the middle and high school level, 90% and 80% respectively filled out math surveys. The response of parents to the survey was surprising. Not only did 56% of the parents return a survey, a percentage much higher than is normally achieved, but many parents wrote

lengthy answers in response to the four open-ended questions and indicated how grateful they were to have their ideas solicited. Seven of the 11 businesses which were selected participated in interviews.

The high response rates on questionnaires may be the result of the assistance of the math representatives at all schools. Because these teachers personally explained the importance of the evaluation, as well as distributed and collected the surveys, school personnel may have felt more involved in and responsible for the evaluation. In addition, parents received surveys from their children rather than through the mail and this may be partially responsible for the good parental response rate. The math representatives and the itinerant math teachers volunteered a lot of their own time for the evaluation and their efforts are in large part responsible for the success of the endeavor. See Table 2 for a summary of the response rates.

Table 2
Response Rates on Math Evaluation

	Elementary	Middle	High
Teachers	76%	82%	79%
Counselors	N/A	74%	75%
Principals	76%	55%	65%
Students	96%	90%	80%
Parents	55%	65%	48%

Description of Respondents

Demographic information was collected from most of the groups who were surveyed. Each group will be described to provide background information that is useful when reading the evaluation report.

Elementary Teachers. A total of 264 teachers responded to the elementary teacher survey. Of these teachers, 11% taught first grade, 12% taught second grade, 10% taught third grade, 9% each taught fourth and fifth grade, and 29% taught a combination of grades. The teachers were quite experienced: 20% had taught school between one and five years, 19% between six and ten years, 24% between 11 and 15 years, 20% between 16 and 20 years, 11% between 21 and 25 years, and 6% had taught school 26 years or more. The average number of years the elementary teachers had been teaching school was 13.6 and most of that teaching experience, an average of 10.8 years, was in APS.

Fifty percent of the elementary teachers reported that the highest degree they had earned was a Bachelor's degree, while 45% had earned a Master's degree, and 4% had earned a Ph.D, an Ed.D, or other advanced degrees. Not suprisingly, the majority (74%) of these teachers held elementary

certification, five percent also held secondary certification, and 21% had elementary certification plus another certificate such as special education or bilingual education.

Secondary Teachers. Two hundred forty-two secondary teachers responded to the math survey; 49% were middle school math teachers and 51% were high school math teachers. Most of these teachers taught a combination of grades at the middle school or high school. The secondary math teachers were almost identical to the elementary teachers in terms of years of teaching experience. The secondary math teachers had an average of 13.3 years of teaching experience, with an average of 11.5 years of this experience in APS. Twenty-one percent of the math teachers who completed surveys held elementary certificates, 69% had secondary certificates, 5% had both, and 2% were certified in other areas. Forty-four percent of the teachers had Bachelor's degrees, 48% had Master's degrees, and 6% had advanced degrees.

Counselors. Ninety counselors, 40% at the middle school level and 60% at the high school level, completed questionnaires on the APS math program. Almost all (84%) of the counselors had attained a Master's degree, 8% had an Education Specialist degree, 3% had doctoral degrees, and 2% had other degrees. The counselors had an average of 13 years experience in counseling with 11 of these years in APS.

Principals. One hundred principals and assistant principals returned surveys about the math program. Fifty percent were elementary principals, 14% were middle school principals, 7% were high school principals, and 26% were assistant principals at all levels. Information on degrees and experience was not requested from principals.

Students. Three hundred seventy-eight fourth grade students completed math attitude surveys. More than half (58%) of these students had attended APS for all five years of their schooling, while 14% had attended for four years and 17% had attended APS three years or less. Five hundred twenty-six students in seventh grade math classes completed surveys and of these 43% had attended APS for eight years, while 16% had attended APS for seven years. Thirty-three percent of the middle school students had attended APS six years or less.

At the high school level, 598 students responded to the math questionnaire. Fifty-four percent of the high school students reported that they had attended APS either 9, 10, or 11 years, while 39% attended APS between one and nine years. Twelve percent of these students were in basic math, 29% were taking algebra I, 41% were enrolled in geometry, and 17% were in algebra II honors.

Because of the way the sample was selected and because most students take just three years of math, very few seniors were represented in the sample. Twenty-nine percent of the students in these four classes were freshmen, 38% were sophomores, 28% were juniors, and 3% were seniors.

Students with a wide range of achievement levels were represented in the sample. Seventeen percent of the high school students reported they had made an A in math on their last report card, 29% had made a B, 25% had made a C, 17% had made a D, and 11% had failed. Interestingly, only 45% of the students believed that their math grade reflected what they'd learned, while 39%

believed it had not, and 16% held no opinion. The majority of these students (69%) said they were college-bound, while 8% said they planned to work, 6% planned to attend vocational school, 6% planned to join the military, and the remainder were undecided or had other plans after they graduated from high school.

Parents. Nearly a thousand parents (965) completed and returned questionnaries. Parents of elementary students accounted for 25% of the responses, while parents of middle and high school students accounted for 38% and 37%, respectively. Most of the parents (38%) indicated they had taken four years of math in high school, 27% reported they had taken three years, 19% had taken two years, and 6% had taken only one year. Although more than half of the parents (52%) did not take math in college, 90% said they enjoyed doing math in daily activities. Eighteen percent reported they had taken one year of college math, 15% had taken two years, and 11% had taken three or four years of college level mathematics in college.

Employers. It was possible to conduct interviews with seven of the 11 businesses which were selected. These were: General Electric Aircraft Engine Group, the Public Service Company of New Mexico, Sandia National Laboratories, Southwest Community Health Services, Sperry Corporation, Sunwest Bank, and the University of New Mexico. Most of the people interviewed were personnel managers or employment representatives.

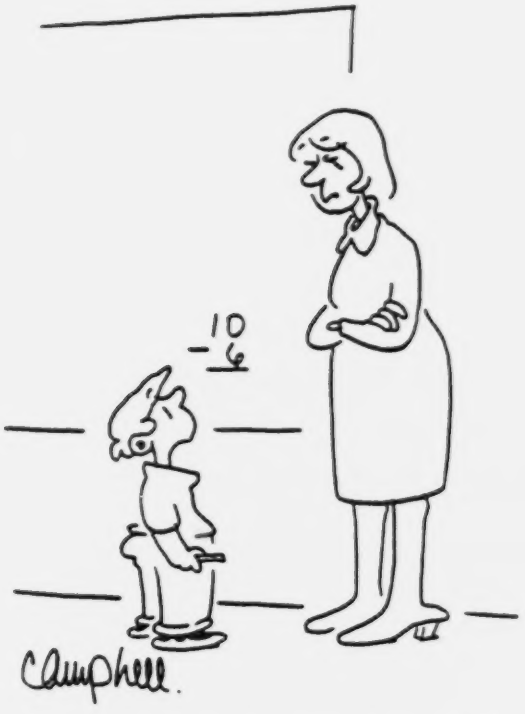
Limitations of the Design

Every evaluation is limited in some way by constraints of time and money. The task of an evaluator is to work within the bounds of available resources to develop the best possible design and to make clear any inherent limitations.

The limitations of the math curriculum evaluation are in two areas. First, information for the math evaluation was collected primarily by questionnaires. While the strengths of this approach are that it allows the collection of data from a larger group of people than would be possible if interviews or observations were used, the limitations are first, people may tell you what they think you want to hear and second, the questions may not cover important areas which might surface in an interview or observation. The latter limitation was addressed by involving many people in the development of the evaluation framework questions and questionnaires and by providing space on all questionnaires for any additional comments or suggestions about the APS math program.

Second, it is never practical to survey the entire population, for example all teachers, all students at every grade level, or all parents. Therefore, the evaluation results are representative only if the sample is selected in such a way that it reflects the total population, and the response rate for the sample is quite high. To address this limitation, the sampling plan was carefully developed by the evaluator and the curriculum coordinator to reflect the APS math program and to use techniques, such as random sampling, which ensure the generalizability of the results. In addition, the math representative in each school was used as an evaluation liaison person to improve the response rates on the questionnaires. This approach was effective and resulted in very good response rates for most groups.

Therefore, although these limitations must be stated they do not present a significant problem in providing a comprehensive, accurate evaluation of the APS mathematics program.



"No, Billy, the difference between 10 and 6 is not a rather gray area."

History of Math Education

Mathematics is one of the oldest of school subjects. Although recorded mathematics began about 2000 B.C., mathematics is more alive and growing today than ever before. To appreciate the growth of mathematics, and thus the changes in mathematics education over the last 141 years, consider that according to the oldest Harvard College catalog housed in its Archives, the algebra that is now taught at the ninth grade and some of the geometry now taught in elementary school and the tenth grade was the extent of the standard mathematics curriculum at Harvard in 1825. By the turn of the century, algebra was still a standard course and the combined content of both the undergraduate and graduate programs did not include all that is now taught in high school. This rapid growth, along with the dual role of mathematics in the school curriculum as a "tool" subject and as a liberal arts subject, has been the root of continual controversy concerning both the content and focus of instruction in mathematics (College Entrance Examination Board, 1985).

Educational Reform Movements

Throughout the history of mathematics education in the United States, two primary issues have persisted: (1) Does school mathematics instruction serve the needs of individual students? (2) Does school mathematics instruction contribute effectively to the solution of the problems of our society? As the students to be educated increased in number and diversity and our society changed from agricultural to industrial to informational, so did mathematics instruction. In the past thirty years, three major reform movements have influenced mathematics instruction. These are new math, back-to-basics, and the information age.

New Math. The new math revolution was sparked by the launch of Sputnik in October, 1957. The movement was initiated to meet the national need for people trained in science and mathematics and was fueled by public decry of students' lack of mathematical skills. This perception persisted despite the fact that the focus of the mathematics curriculum of the 1940's and 1950's was virtually all skills (Reeve, 1955). The hallmarks of the new math movement were discovery, rigorous logic, and structure, and the major objective was to increase skills as well as understanding.

Studies such as the National Longitudinal Study of Mathematical Abilities (NLSMA) and the National Assessment of Educational Progress demonstrate that the new math met its objectives (Carpenter, Lindquist, Matthews, & Silver, 1975). From such studies, it is apparent that students who benefited most from the new math curriculum were the better students and that declines in test scores in the late 1960's and early 1970's were due primarily to the increasing numbers of students taking the tests (Wirtz, et al., 1977). Later declines in the scores on the mathematics portion of the Scholastic Aptitude Test (SAT) have been correlated with steep declines in SAT verbal scores and differences in items tested (Wirtz, et al., 1977).

Back-to-Basics. Regardless of the reasons, test scores continued to decline and the general public blamed this on the curriculum, which resulted in the "back-to-basics" movement. This movement, still strong in many parts

of the country, emphasized paper-and-pencil computational skills, especially at the elementary school level. In addition, minimum competency testing was established in many states and school districts to make decisions about promotion or graduation. Generally, the percentage of students who fail these tests the first year they are given is quite high, but decreases in succeeding years with remedial efforts and changes in instructional materials and teaching strategies. The most recent NAEP (1983) results indicate increasing performance among black and Hispanic youth at all ages tested, among all 13-year-olds, and among the bottom 25% of students.

The cost of these successes has been great. Whereas in the 1960's, most students were subjected to a curriculum that addressed the better students, most students in the 1970's were subjected to a curriculum appropriate for the less skilled students. The performance of the top 25% of students on the NAEP declined and general mathematics achievement declined as well, especially in problem-solving and understanding of concepts (Carpenter, et al., 1983).

The Information Age. The changing economic status of the United States in the world market, the influence of technology on all facets of society, and the changing structure of the family in the United States are among the forces that precipitated the most recent demands for changes in mathematics instruction. Within this decade, a variety of government and privately-sponsored reports have recommended changes in mathematics education. Now, as thirty years ago, there is dissatisfaction with student performance. Mathematical skill is portrayed as vital to economic well-being, to the country's future strength, and to the ability of citizens to participate fully in the economy of the information age. In the 1950's, the challenges were space and military competition with the Soviet Union. Now, are added the challenges of computers and economic competition with Japan and Western Europe.

The most important single attribute of the Information Age economy is that it represents a profound switch from physical energy to brain power as the driving force, and from concrete products to abstractions as the primary products. Instead of training all but a few children to function smoothly in the mechanical systems of factories, adults who can think are needed. This is significantly different from the concept of an intellectual elite having the responsibility for innovation, while workers take care of production (Zarinnia & Romberg, in press).

Basic communication skills are more important than ever before, necessitating a literacy-intensive society. Information only has value if it can be controlled and organized for a purpose. To tap the power of computers, it is obligatory to first be able to communicate efficiently and effectively, that means being both literate and numerate. In addition, in an environment of accelerated change, the old approach of training for a lifetime occupation must be replaced by "learning power," which also depends on the ability to understand and to communicate (Naisbitt, 1982).

American schools, like public schools in other industrialized countries, have inherited two quite distinct educational traditions-one aimed at the education of an elite, the other concerned with mass education. These traditions conceived of schooling in different terms, had different clienteles, and held different goals for their students. Despite a gradual merging of the two

traditions in American schools, there is a continuing and, as yet, unresolved tension between the goals and methods of elite education and mass education that is producing our current concern for the teaching of higher order skills (Resnick, in press).

The Changing Role of Mathematics in Society

Focusing instruction in mathematics on higher order thinking skills is a complete turn-around from the focus of the back-to-basics movement, as identified by Lauren Resnick (in press) in a paper, prepared for the National Research Council. Higher order thinking:

- is non-algorithmic. That is, the path of action is not fully specified in advance.
- tends to be complex. The total path is not "visible" (mentally speaking) from any single vantage point.
- often yields multiple solutions, each with cost and benefits, rather than unique solutions.
- involves judgement and interpretation.
- involves the application of multiple criteria, which sometimes conflict with one another.
- often involves uncertainty. Not everything is known that bears on the task at hand.
- involves self-regulation of the thinking process. We do not recognize higher order thinking in an individual when someone else "calls the plays" at every step.
- involves imposing meaning and finding structure in apparent disorder.
- is effortful. There is considerable mental work involved in the kinds of elaborations and judgements that are required.
- skills cannot be monitored by the three contemporary approaches to achievement testing (standardized tests, profile achievement tests, and objective referenced tests).
- skills are not to be learned after other skills.

The most important single message of modern research on the nature of thinking is that the kinds of activities traditionally associated with thinking are not limited to advanced levels of development. Instead, they are an intimate part of even elementary levels of mathematics and other branches of learning, when they are proceeding well. In fact, the term "higher order" skills is probably itself fundamentally misleading, for it suggests that there is another set of skills, presumably called "lower order," that needs to come first. This assumption, that there is a sequence from lower level activities that do not require much independent thinking or judgment to higher level ones that do, colors much educational theory and practice. Implicity, at least, it justifies long years of drill on the "basics" before thinking and problem solving are attended to or demanded. A fundamental challenge to this assumption is provided by cognitive research on the nature of basic skills such as mathematics. Mathematics education is not adequately engaging students' capacities for interpretation and constructing meaning. Mathematics is taught and learned by most students as routine skills, while higher order capacities for organizing and interpreting information often are not exercised or developed.

Since the middle 1970's, the mathematics community has recognized and addressed the need for change in mathematics education. An Agenda for Action,

a widely distributed document developed by the National Council of Teachers of Mathematics (1980) combined many of the recommendations of its predecessors. Four of its eight recommendations deal directly with curriculum:

1. Problem solving should be the focus of school mathematics in the 1980s;
2. Basic skills in mathematics should be defined to encompass more than computational facility;
3. Mathematics programs should take full advantage of the power of calculators and computers at all grade levels; and
4. More mathematics study should be required for all students and a flexible curriculum with a greater range of options be designed to accommodate the diverse needs of the student population.

These recommendations reflect the growing application of mathematics in every facet of today's highly technical society. Not only do science-related fields rely heavily on mathematics, but advanced mathematics is now required for studying medicine, architecture, business administration, forestry, accounting, and many other fields. In addition, this generation has seen the sudden emergence of computer technology, which draws on the full spectrum of mathematics. While reducing much of the drudgery of life, computers have increased the amount of mathematical knowledge and facility required to live in a computerized world. People use some application of mathematics every day and continually rely on a multitude of human services without ever realizing that those services are made possible through the power of mathematics.

The California State Department of Education (1982) states:

> To prepare students for life in today's highly technical society, their mathematical training must include and go far beyond providing training in the simple skills of counting, computing, putting numbers into formulas, and even solving equations. Learning only rote mathematical rules ill equips students to apply those rules to solve problems outside the classroom. Instead, the mathematics curriculum must focus on what mathematical concepts mean, how they are related, and where they apply. Most importantly, all mathematics concepts must be taught in such a way that students understand their application in day-to-day living and their value in various careers and vocations. (p. 1)

Implications for Change

Reports from both the mathematical education community and from outside agree almost unanimously that two simultaneous changes must occur in mathematics education. First, student performance must be upgraded. Requirements for promotion and graduation should be increased and emphasis should be shifted from rote manipulation to problem solving, with increased student achievement in both areas. Second, the curriculum, teaching strategies, and methods of assessing achievement should be updated to reflect the overall objectives of math education.

At first glance these changes appear not too difficult to implement. They suggest that we must simply require higher grading standards, add more content to the curriculum to meet new objectives, and require more mathematics for high school graduation. However, the complexity of implementation stems from diverse sources, particularly from the fact that these two changes are not disjoint; each is dependent upon the other.

First, how mathematics is taught and assessed influences student performance, and student performance at the elementary and middle grade levels influences the content of the high school curriculum. Second, student performance is directly related to the preparation of teachers. Many teachers without sufficient mathematics training are being asked to fill vacancies in mathematics classrooms. The growing shortage of qualified teachers of mathematics in the United States has placed the quality of mathematics instruction for our youth in jeopardy (National Commission on Excellence in Education, 1983). Third, the recommended changes require funding. Additional course requirements for high school graduation necessitate more teachers qualified to teach the course. Hiring new teachers, retraining teachers displaced from other academic fields, and providing professional development opportunities for mathematics teachers to learn the recommended new curriculum content and teaching strategies puts additional financial burdens on currently limited resources. Fourth, results of the recommended changes take time. Because of the sequential nature of mathematics, change does not occur piecemeal. To be effective, an entire generation of students must participate in the change. Finally, how mathematics is taught and assessed is influenced by a variety of sources, which are not in agreement as to the objectives of mathematics education.

The current national concern over mathematics education has provided impetus for change and many new programs have already shown positive results. A report by the Education Commission of the States (1984) found that 44 states either had raised high school graduation requirements or had such changes pending. Twenty-seven states had enacted new policies dealing with instructional time, such as new extracurricular policies and reduced class sizes. But what will such changes actually mean in the classroom? New course requirements do not necessarily deal with the academic quality of the courses used to fulfill those requirements. Concerns about the curriculum and the evaluation of student performance still exist.

Evaluating Student Performance

Evaluation of student performance by teacher-made tests, competency tests, or norm-referenced tests should reflect the priorities identified for mathematics teaching and learning. All tests should be selected or designed to be compatible with curriculum priorities. While it is easier to assess performance on lower order computational skills than on higher order analysis and problem-solving skills, every effort should be made to maintain the same balance in tests that exists in the curriculum. Furthermore, evaluation should be based on classroom observations and discussions as well as on test results.

The College Entrance Examination Board (1985) stated in its publication on mathematics preparation:

> We do not wish to add to teachers' burdens, nor are we under any illusions about the ease of making continuous assessments. But too great an emphasis on multiple-choice and short-answer testing is in part responsible for the fragmented curriculum in school mathematics today. If we are to refocus the curriculum on a smaller number of topics treated in greater depth, the assessment process needs to be refocused as well. (p. 78-79).

Student success in mathematics depends not only on good curriculum, but also on good teaching. Teachers should possess a thorough knowledge of the mathematics being taught, understand why the mathematics is being taught, and be able to select from a variety of instructional strategies for each concept that is being taught. Since student achievement in mathematics depends to a large degree on the amount of instruction in mathematics, students at all grade levels should receive 50 minutes or more of mathematics instruction per day. Teachers should maximize the time each student is successfully engaged in learning mathematics (Wisconsin Department of Public Instruction, 1986).

The K-8 Curriculum

Unless the school year is lengthened as recommended in A Nation at Risk, (National Commission on Excellence, 1983) or unless time is taken from other school activities, it is impossible to meet added expectations in mathematics without eliminating some of the traditional content or revising the curriculum and teaching strategies to more effectively combine content. In its "Position Paper on Basic Skills", the National Council of Supervisors of Mathematics (1978) recommends specific changes in priorities and emphases in the instructional program to reflect the expanded concept of basic skills. Among these are increased emphasis on activities such as:

- locating and processing quantitative information;
- collecting, organizing, presenting, and interpreting data;
- estimating measures and mentally estimating results of calculations;
- using imagery, maps, sketches, and diagrams as aids to visualizing and conceptualizing a problem;
- using concrete representations, including manipulatives, to model numeric and spatial relationships;
- using technological aids to calculate.

There should be decreased emphasis on activities such as:

- isolated drill with numbers apart from problem contexts;
- performing paper-and-pencil calculations with numbers of more than two digits.

Each year since the publication of these recommendations, more states and districts have incorporated them into their own standards for mathematics instruction. Among the latest to do so is the Wisconsin Department of Public Instruction (1983), whose A Guide to Curriculum Planning in Mathematics includes specific classroom implementation of these recommendations. They state:

> Facility with one-digit number facts is as important as ever, but drill on the arithmetic operations with larger than three-digit numbers should be de-emphasized. Such computations can and should be done by calculators and computers. The selective use of calculators and computers should be encouraged, both to help develop concepts and to do many of the tedious computations that previously had to be done using paper and pencil. (p. 17-18).

Students need to develop a feel for the magnitude of numbers and understand the relationship of numbers to physical situations that they represent. Appropriate manipulatives should be used to give meaning to algorithms. When

using manipulatives, the correspondence between the physical manipulations and the written algorithm should be stressed. Students should also learn to estimate an answer and should regularly use estimation to check the reasonableness of their answers.

Working with numbers and number operations should not be carried out in the abstract. Operating within a real-world, problem-solving context gives meaning to numbers and to the operations on them. This is particularly important for fractions and decimals. The need for non-whole numbers should arise within a measurement context so that children can come to realize that fractions and decimals are, indeed, numbers that behave in much the same way as whole numbers. The understanding of fractions and decimals as numbers should receive more emphasis, while drill on addition, subtraction, multiplication, and division of fractions and decimals should be given less emphasis.

Mental arithmetic should be emphasized at all levels, first aimed at exact answers and later at approximate ones. Mental arithmetic involves finding natural and easy, not formal algorithmic, ways of looking at number relationships. There should be heavy and continuing emphasis on estimation, not only in formal round-off procedures, but in developing a feel for numbers. Students need experience in estimating actual quantities as well as in estimating numerical quantities that appear in complicated form.

Calculators should be used to develop concepts as well as eliminate tedious computation. With calculators students can solve problems before they have mastered computation skills. This helps to provide meaning for the operations. Calculators also make it possible to use realistic numbers in problems. They are also appropriate tools to use in developing concepts like factorization or square root.

Problem solving integrated throughout the curriculum and instruction should emphasize the application of arithmetic skills to solve problems. The interrelationships between the different strands of mathematics should be made apparent. Problem situations provide meaning for arithmetic operations, and instruction can portray arithmetic as a system of representing and solving a variety of types of problems. Applications should always stress good problem-solving techniques, not just guessing which arithmetic operations should be used. The importance of different real-world applications of mathematics will change with time. Applications should be designed to develop general skills for applying mathematics to new situations, not to drill on particular applications that are likely to become outmoded.

Enrichment should involve the use of arithmetic in applied situations, not additional practice in computation. More capable students should be encouraged to develop their understanding of basic concepts and their problem-solving abilities, not their computational proficiency. Decisions about grouping for enrichment should not be made strictly on the basis of computational skills (Wisconsin Department of Public Instruction, 1983).

The High School Curriculum

Over the past few years, three factors have most conspicuously affected the high school mathematics curriculum: widespread increased requirements for graduation, increasing numbers of students who pursue post-secondary studies, and growing pressure to include new content.

The traditional structure of the first three years of mathematics in high school for college preparatory students has been two years of algebra and one year of geometry. For students who are deemed unprepared for or incapable of the study of algebra, alternatives such as general mathematics, consumer mathematics, business arithmetic, and mathematics for various vocational subjects has been offered. With the need for advanced preparation in mathematics for entry into a growing number of fields and for post-secondary study, there is a growing emphasis for all students to receive essentially the same basic preparation in mathematics, with the principal variation being in the rate at which the arithmetic is presented (College Entrance Examination Board, 1985).

The Conference Board of the Mathematical Sciences (1983) recommends that:

- The traditional component of the secondary school curriculum be streamlined to make room for important new topics. The content emphases, and approaches of courses in algebra, geometry, precalculus, and trigonometry need to be re-examined in light of new computer technologies.

- Discrete mathematics, statistics and probability, and computer science now be regarded as "fundamental" and that appropriate topics and techniques from these subjects be introduced into the curriculum. Computer programming should be included at least for college-bound students.

These recommendations are currently being followed in a variety of programs. Most notable of these are state-developed programs in New York, Oregon, California, Ohio, and Wisconsin, the University of Chicago School Mathematics Project, and the Computational Problem Solving program recommended by the College Entrance Examination Board. The common ingredient in these programs is the integration of arithmetic, algebra, geometry, trigonometry, probability and statistics, and logic into a comprehensive three-year program with emphasis on reading, writing, speaking, listening, and reasoning.

Summary

The history of mathematics education is a study in change. However, never before has the change been as universally demanded, as extensively researched, or as comprehensive. The recommendations made by many professional math educators and cited in this report have widespread effects. They cause students to have higher expectations for more and different mathematics courses. They require teachers to gain knowledge of new content, new approaches to teaching the content, and new ways of monitoring student performance. They require teacher training institutions to offer more and better training and retraining opportunities for teachers. They require local school districts to provide ongoing inservice and staff development for mathematics teachers. They require state and local boards of education to provide philosophical and financial support. They require state departments of education to provide leadership and assistance. And they require all members of the community to recognize the importance of mathematics and to provide support for quality mathematics instruction for all students.

Goals

The APS mathematics program, rather than having formalized goals, specifies key competencies students should master in grades one through eight and delineates course objectives for high school mathematics classes. In order to address the area of mathematics goals, two questions were asked of teachers, counselors, principals, and parents. First, they were asked to indicate who or what should be most influential in determining the goals of the APS math program. Second, they were asked to indicate the importance of several general goals for mathematics education.

All of the groups surveyed were unanimous in indicating that the classroom teacher should be the most involved in determining the goals of the APS math program, followed by the district math specialists. It was believed that parents should be the next most involved in determining the goals, and next, public school administrators. Although the requirements of the post-secondary schools were seen as the fifth most important in determining math goals, rankings varied from a high of second place by parents to a low of tenth place by secondary teachers. The involvement of the State Department of Education, professional math organizations, students, employers, accrediting associations, and the state legislature were considered to be less important in determining math goals, with the legislature ranked lowest by all groups. See Table 3.

Table 3
Recommended Involvement of Various Groups in
Determining APS Math Goals

	Ranking By:					
	Elementary Teachers	Secondary Teachers	Counselors	Principals	Parents	Overall
Classroom Teachers	1	1	1	1	1	1
District Math Specialists	2	2	2	2	2	2
Parents	4	3	5.5	4	4.5	3
Administrators	5	6	3	3	7	4
Post-Secondary Schools	3	10	4	7	2	5
State Department of Education	6	8.5	8	5	5	6
Professional Math Organizations	7	4	5.5	9	9	7
Students	8	7	10	6	6	8
Employers	10	5	7	8	10	9
Accrediting Associations	9	8.5	9	10	8	10
State Legislature	11	11	11	11	11	11

Teachers, counselors, principals, parents, and high school students felt the most important goal for a mathematics program was to teach basic mathematical knowledge. The second most important goal was believed to be preparing students for further education. The third most important goal was thought to be enabling students to use math in their daily living. The goal ranked fourth most important was teaching mathematical thinking and problem solving,

followed in fifth place, by preparing students for jobs. Encouraging students to enjoy math for its own sake and to enjoy math in leisure time were ranked in sixth and seventh places, although these goals still were considered to be somewhat important. See Table 4.

Table 4
Rankings of Importance of
Mathematics Goals

	Elementary Teachers	Secondary Teachers	Counselors	Principals	Parents	High School Students	Overall
To teach basic mathematical knowledge	1	1	1	1	2	2	1
To prepare students for further education	3	2	2	3	1	1	2
To be able to use math in daily living	2	3	3	2	3	3	3
To teach mathematical thinking and problem solving	4	4	4.5	4	5	5	4
To prepare students for jobs	5	5	4.5	5	4	4	5
To encourage students to enjoy math for its own sake	6	6	6	6	6	6	6
To teach students to enjoy math in their leisure time	7	7	7	7	7	7	7

"I realize it has no value, but I strongly believe in basic research."

Content

Perhaps more than any other curricular area, the content of mathematics as a field is relatively easy to define. Although there may be disagreement about the sequence in which concepts are presented or the age at which it is appropropriate to introduce concepts, for the most part a person can describe what math is. However, the agreement ends there. Studies indicate that a national math curriculum does not exist. Although professional math organizations have recently developed guidelines for the content of the mathematics curriculum (CBMS, 1983; National Council of Supervisors of Mathematics, 1978; NCTM, 1980, 1981), studies show there is a great deal of discrepancy among textbooks in terms of content as well as among what is measured by standardized tests (Freeman et al., 1983; Raizen & Jones, 1985; Trafton, 1980). In addition, teachers may stress different subject matter regardless of the textbook being used (Raizen & Jones, 1985). Thus, discovering how the content of a math program is determined is of special importance.

With an increasing public emphasis on accountability in the teaching profession, the views of parents and employers are considered to be important in determining what should be taught in our schools (Raizen & Jones, 1985; Trafton, 1980). Therefore, the opinions of these groups regarding the content of the math program also are of importance in a curriculum evaluation. In addition, the adequacy of the curriculum in providing students with varying abilities and goals the necessary math skills is important (Bell, Costello, & Kuchemann, 1983).

This section of the report will address issues related to the content of the APS mathematics program at the elementary, middle, and high school levels, including:

- how the content of the APS math program is determined and how various groups believe the content should be determined
- the familiarity of various groups with the math program
- the adequacy of communication about the math program
- the effects of standardization on the math curriculum
- the adequacy of the scope of the math program
- high school mathematics requirements
- restructuring the secondary math program.

Determining the Content of the Math Program

The majority of elementary and secondary teachers reported that they occasionally refer to the materials provided by the district, such as key competencies, teacher edition insert sheets, and yellow book outlines, to guide what they teach in mathematics. Elementary teachers were slightly more likely than secondary teachers to use these materials. While about the same percentage of teachers at both levels (9% at the elementary level and 8% at the secondary level) indicated they used these materials often, more secondary teachers (15%) than elementary teachers (9%) indicated that they never refer to district-provided resources. This does not necessarily mean that the guidelines are not being followed. It might mean that teachers are familiar

with the resources and don't need to refer to them or perhaps that they don't find the resources to be helpful. If district-provided resources are not frequently used to guide the teaching of mathematics, how do teachers feel the content of the math program should be determined?

All of the groups surveyed including teachers, counselors, principals, and parents were unanimous in their belief that the classroom teacher should be the most involved in determining the content of the APS math program. This was closely followed by district math specialists who were ranked second most important in determining math content. Although there was more discrepancy among the various groups for the third place ranking, post-secondary schools were rated overall as the third most important determinant of the math program. The content of the textbook was also deemed important with an overall ranking of fourth place, but there was much disagreement among groups as to how much influence the textbook should have. Responses ranged from a second place ranking by parents to a ninth place ranking, out of 12 choices, by principals.

Administrators were ranked fifth overall in recommended involvement in determining math content, varying from a high of third place by counselors and principals to a low of eighth place by secondary teachers. This low ranking by secondary teachers may be due to their specialized math knowledge and a perception that principals cannot be knowledgeable in all of the subject areas taught in middle and high school.

Parents were ranked sixth, followed by professional math organizations, the State Department of Education, students, accrediting associations, and employers. In last place was the state legislature, and the consensus was unanimous that they should be the least involved in determining the content of the APS math program. See Table 5.

Table 5
Recommended Involvement in
Determining Math Content

	Elementary Teachers	Secondary Teachers	Counselors	Principals	Parents	Overall
Classroom Teachers	1	1	1	1	1	1
District Math Specialists	2	2	2	2	3	2
Post-Secondary Schools	4	3	4	6	4	3
Math Textbook	3	4	5	9	2	4
Administrators	6	8	3	3	7	5
Parents	5	6	9	5	6	6
Professional Math Organizations	7	5	6	7	8	7
State Department of Education	8	9	8	4	5	8
Students	9	10	10	8	10	9
Accrediting Associations	10	11	7	11	9	10
Employers	11	7	11	10	11	11
State Legislature	12	12	12	12	12	12

In summary, classroom teachers, district math specialists, post-secondary schools, and math textbooks were believed to be the most important in determining the content of the APS math program. Of moderate importance were administrators, parents, professional math organizations, and the State Department of Education. Those who should be least involved in determining the math curriculum were thought to be students, accrediting associations, employers, and the state legislature.

Familiarity With the Math Program

As might be expected, classroom teachers reported that they are most familiar with the content of the math program at the level at which they teach. But, elementary teachers are more familiar with the math content at their own level than are secondary teachers. On the other hand, secondary teachers are more generally knowledgeable about the content of the math program at all three levels, while elementary teachers are comparatively much less familiar with the content of the middle and high school programs. Principals, counselors, and parents reported moderate familiarity with the math program at all levels. Parents, however, were successively less familiar with the math program at each higher level in spite of the fact that twice as many secondary parents as elementary parents responded to the survey. The results as a whole are not surprising in light of the increasing complexity of the math program at higher levels and the likelihood that most respondents have had elementary mathematics while fewer have taken advanced mathematics.

Communication with Parents

Parents also were asked how they received most of their information about the APS math program, how adequate they felt communication about math was, and how communication might be improved. Forty-two percent of parents with students who were entering middle and high school the following fall said they were somewhat familiar with the math classes that were offered. Fourteen percent said they were very familiar, while fifteen percent indicated they were not at all familiar. In spite of this group's relative familiarity with secondary math offerings, 35% of the parents felt the communication about math classes between home and school was inadequate. Forty-five percent felt communication was somewhat adequate while 11% thought it was very adequate. Parents reported that they learned the most about the APS math program from their child's teacher, and the next most from friends or neighbors. These were followed by an almost equal reliance on school or district newsletters and the city newspaper. The least number of parents said they learned about the math program through special activities, such as workshops for parents.

When asked how communication with parents about math could be improved, 545 open-ended responses were made. At the elementary level the greatest number of suggestions had to do with providing parents information about the math skills students would be expected to master that year. Some parents felt an outline at the beginning of the year would be sufficient, while others thought information every quarter or every few weeks would be more helpful. The second largest category of responses was communicating regularly with parents through notes, newsletters, progress reports, and sending home samples of students' work. Some parents also mentioned that they wanted to receive early notification of problems and to be required to sign notices that are sent home. The next most frequent suggestion was for parent-teacher conferences, meetings, or phone calls regarding children's progress in mathematics. Some

parents also believed students should have more homework and that assignments which stressed parents and children working together would be especially helpful. Some parents indicated they didn't feel able to help their children in math because the subject was taught so differently than when they were in school. They felt courses, seminars, or workshops for parents would be very helpful. A few parents said that more frequent communication about math would be helpful, while others emphasized that it was the parents' responsibility to become involved in and knowledgeable about their childrens' schoolwork. A small number of parents thought better textbooks and materials or availability of workbooks for home use would be beneficial. Finally, several parents indicated that they thought the schools or their children's teachers were already doing a good job communicating.

At the middle school level, the majority of parents wanted communication from teachers in three major ways: through personal meetings such as conferences or phone calls; through newsletters, notes, or progress reports; or through the receipt at the beginning of the year or regularly throughout the year, information on course content, goals, and expectations. In addition, some parents said they'd like to receive a general overview of the skills students were expected to achieve at various grade levels. Another small group of parents said more frequent communication would be helpful, while some felt that parents, rather than teachers, should take the initiative to improve communication. Some parents thought that seminars or workshops for parents would be beneficial, especially if there was an opportunity to work some of the math problems their children were currently studying. Along the same line, a few parents felt that joint student-parent assignments would be useful. Middle school parents also desired early notification if their children were having trouble in math.

The high school parents made similar suggestions to those of the middle and elementary school parents. These parents stressed that they would like individual parent-teacher conferences at the high school level, not just group meetings. Many felt that newsletters, notes, or progress reports would suffice especially if they were frequent, contained information about course expectations, and required parent signatures. Like parents at the other grade levels, they felt any communication with the home was adequate as long as it was direct and occurred routinely. A few high school parents also felt they didn't understand their children's work well enough to be able to assist them and recommended workshops or seminars for parents. Some parents felt communication was fine or wasn't necessary as long as the teachers and students were communicating well. A few parents thought parent-student activities would be helpful and some emphasized the parents' responsibility for improving communication by being involved and taking the initiative.

Communication with Employers

All of the businesses reported that they were involved in some way in the public schools. Some businesses sponsored job fairs and science fairs, judged academic competitions, or sponsored a school through the "join-a-school" program. Mentorship programs, the Career Guidance Institute, grants, and presentations to students also were mentioned as examples of their involvement with public schools. In spite of this active involvement with the schools, six of the seven employers indicated that they did not receive information about the APS mathematics program. One business representative mentioned

seeing the summer math calendar and another indicated that information was received from a school board member, from the Occupational Education Committee, from the State Department of Education, and from being a member of a curriculum review team.

Standardization of Curriculum

In recent years an attempt has been made to standardize the math curriculum in APS by providing specific guidelines to teachers and by limiting the number of math textbooks in use. Since many children in APS move during the school year, it was believed that standardization would help both teachers and students cope with this mobility. While standardization may increase the likelihood that students learn the same material in the same order, it can limit the teacher's ability to individualize the curriculum. For this reason, it is important to ascertain educators' professional opinions on the effects of standardization. The majority of the teachers, counselors, and principals who responded to this question felt that standardization had improved the math program. Thirty-nine percent of elementary teachers, 40% of secondary teachers, 42% of counselors, and 66% of principals thought standardization had made the curriculum better. Some of those individuals surveyed felt standardization had made the math program worse-16% of elementary and secondary teachers, 11% of counselors, and 9% of principals. A few of the educators believed that standardization had not influenced the math program either positively or negatively-10% of elementary teachers, 22% of secondary teachers, 2% of counselors, and 7% of principals. The remainder had no opinion on the effects of standardization.

Responses to the preferred method for textbook selection confirmed these results. Generally, teachers, principals, and counselors preferred the current method of textbook adoption-choosing from two or three textbooks that are being used district-wide. The most restrictive adoption model, every teacher using the same text for a particular level or course, was least preferred out of six choices by elementary teachers, ranked third place out of four choices by secondary teachers, but ranked second place out of four choices by principals and counselors. However, the most permissive adoption model, selecting any text from the state approved list, was ranked low or in last place by all groups.

Teachers, principals, and counselors for the most part, then, consider standardization of curriculum to be of benefit to the APS math program.

Adequacy of the Scope of the Math Program

Teachers were asked to evaluate the amount of time currently devoted to major topics in the APS math curriculum. Teachers indicated that approximately the same amount of time or slightly more should be devoted to most topics. However, both elementary and secondary teachers thought a few areas should receive additional emphasis. The area recommended by the most teachers as needing more attention was problem-solving skills and strategies. In addition, some parents commented that the math program should emphasize problem-solving and thinking skills. This is in accord with a finding of the Priorities in School Mathematics (PRISM) study in which teachers, curriculum supervisors, and other educators indicated that problem solving should receive more emphasis (NCTM, 1981).

The second area rated by APS teachers as needing more emphasis was mental

calculations; followed by consumer math skills; organizing, presenting, and interpreting data; drawing inferences and predicting; using logic; classification and properties of geometric shapes; computer use; and at the secondary level computation of fractions and decimals. Parents at all levels thought the APS math program should emphasize the skills needed for daily living and that students should be able to understand the applications of mathematics. Except for the emphasis on computation, these recommendations are consistent with recommendations made by the NCTM (1980) in their publication, An Agenda for Action and others (CBMS, 1983; Driscoll, 1980, 1982; Trafton, 1984). The National Assessment of Educational Progress (1983) indicates that the computation skills of students have improved since the early 1970's, but students have not made corresponding gains in the ability to analyze mathematical problems or apply mathematics to nonroutine situations.

Teachers and principals also were asked what they thought should be emphasized if there were an additional 15 minutes a day or class period to teach mathematics. Interestingly, principals and teachers at both levels responded similarly. The most important of the six areas was considered to be developing and strengthening concepts. This was followed by applying mathematics and solving problems. The fourth most important emphasis was considered to be drilling on the basics. Although a greater percentage of elementary teachers and principals selected this as more important than developing and strengthening concepts, the high percentage of those who indicated drill was least important, lowered the overall ranking of this item to fourth place. The last two topics chosen were building an intuitive base for mathematics and exploring enrichment topics. Like teachers and principals, parents were divided in their opinions on how much time should be spent on the basics. While some parents felt the most time should be devoted to the basics, others felt that too much time was spent on basic skills with a consequent lack of development of understanding and ability to apply mathematics. On a similar question in the PRISM study elementary teachers selected problem solving as most important, followed by application of mathematics and drilling on the basics. These teachers, like APS teachers and principals, ranked enrichment topics and building an intuitive base for mathematics as least important.

Meeting the Needs of Students. School personnel were asked how well they thought the APS math program met the needs of particular groups of students. It was the general consensus of teachers and counselors that the math program met the needs of female, minority, and mathematically talented students, but was not meeting the needs of English as a Second Language (ESL) students or developmental/remedial students. Principals thought the math program was doing alright in meeting the needs of all of these groups of students.

Parents were asked if their child's math class met his or her special needs and also if there were any additional courses or extracurricular math activities they'd like to have offered by the schools. The majority of the parents at all three levels responded that they thought the math program was adequate, that their child was doing fine in mathematics, or that their child's teacher was good. Of those parents who believed math classes were not meeting the needs of their children, the most frequent complaint was that the class was not sufficiently individualized. Some parents indicated that their children needed more enriched math activities, that the classes were geared too much to the slow learners and their children were bored. Other parents, whose children were having difficulty with mathematics, indicated that their

children needed additional help from the teacher or tutoring. Several of the parents commented on the large class size and the lack of time teachers had to individualize math instruction, especially at the high school level. A few parents believed that mathematics was taught in a rote fashion, with not enough emphasis placed on understanding mathematics or its applications to real life.

Of the parents who indicated they would like their child's school to offer an additional course or extracurricular activity, the most frequently made suggestion was for a computer class or club, followed by individualized math clases, either remedial or enriched in nature. In addition, some parents desired a focus on making math fun, challenging, and meaningful through contests, clubs, games, and a positive teacher attitude.

In summary, although for the most part, school personnel and parents believed the math program met the special needs of students, the areas of concern seem to be meeting the needs of slow learners and ESL students. In addition, some parents didn't believe sufficient enrichment activities were being provided in regular math classes, although school personnel believed the needs of the mathematically talented were well met. This may be due to lack of parent knowledge about special math opportunities or programs or there may be a need for enrichment activities for some students who are not in special programs such as the gifted program.

Integrating Math into Other Subjects. School personnel thought it was more important than did parents to integrate math into other subjects at all grade levels; however, parents felt it was moderately important. With the exception of secondary teachers, all others surveyed including elementary teachers, principals, counselors, and parents felt it was most important to integrate math into other subjects at the elementary level, followed by the middle school level and high school level. Secondary teachers thought it was most important to integrate math at the middle school level, then at the high school level, and last at the elementary school level.

High School Mathematics Requirements

With the passage of educational reform acts in many states, emphasis on the importance of mathematics has increased. State legislatures are now mandating increased graduation requirements and sometimes specifying in what subjects these requirements must be met. Therefore, it was considered important to elicit opinions on recommended math requirements for high school students. To find out the importance of various math classes to the high school program, secondary teachers, principals, and counselors were asked to indicate which math course they would require of all high school students. Algebra was the course most often designated as a requirement. Forty-six percent of secondary math teachers, 32% of principals, and 43% of counselors who responded to the question, selected algebra. Consumer or business math, which was described as giving students the skills to use math in their everyday lives, was the recommended required course by the second largest group. Twenty-seven percent of math teachers, 32% of principals, and 29% of counselors indicated they would require this course. The third most frequently recommended required math course was basic or general math with 6% of math teachers, 23% of principals, and 20% of counselors indicating this was important. Other courses such as geometry or individualized math review were recommended by only a few people.

These same groups also were asked when they thought the three required years of mathematics should be taken. The largest percentage of each group recommended the current approach which is one math course in ninth, tenth, and eleventh grades. Thirty-nine percent of math teachers, 40% of principals, 59% of counselors, and 39% of parents recommended this approach. The second most frequent response was that the years in which math courses are taken should be the student's choice, with 29% of teachers, 19% of principals, 18% of counselors, and 24% of parents selecting this answer. The third choice was for students to take math in the tenth, eleventh, and twelfth grades - 12% of teachers, 15% of principals, 14% of counselors, and 20% of parents selected this option. The least frequently chosen responses were a combination of grades with no math course taken between two of the grades. Clearly, APS personnel and parents prefer the current approach, although taking math closer to the time of graduation might improve students' test scores on college entrance or employment exams.

High school students were asked how many years of mathematics they preferred to take. Contrary to what might be expected, 51% of the high school students who responded to the survey said they preferred to take four years of mathematics in high school. Twenty-one percent preferred three years, 18% preferred two years, 6% preferred one year, and 5% preferred to take no math. Although 38% of students polled said math was one of their two favorite subjects in high school, some of this "preference" for math may be due to students' realization of the importance of math to their futures.

Restructuring the Secondary Math Program

For years, the high school math program has been geared towards preparing college bound students for calculus, in spite of the fact that only 10% to 20% of all high school students pursue math-related careers requiring calculus (Sonnabend, 1985). Non-college bound students, on the other hand, typically have studied general and consumer mathematics which has not adequately prepared them for their futures (Chambers & Kepner, 1985; Sonnabend, 1985). Some math educators believe that the entire high school math program must be restructured to solve the complex problems of inadequate student preparation in mathematics (Chambers & Kepner, 1985; Driscoll, 1982; NCTM, 1980; Sonnabend, 1985; Usiskin, 1983, 1985). Usiskin (1983) recommended a broadening of the program for non-college-bound students to include: a year of consumer mathematics and one semester each of applied algebra, applied geometry, computers, and statistics. Sonnabend (1985) believed the math program for college-bound students also must be changed to emphasize applications and problem-solving and incorporate the study of non-career mathematics, computers, statistics, and probability. Some educators believed that the traditional algebra I, geometry, algebra II, calculus sequence is no longer adequate and an integrated math program would be more appropriate (Manhard, 1985).

Much controversy surrounds the structure of the high school math program including when algebra and calculus should be taught, how algebra and geometry should be sequenced, if algebra and geometry should be integrated into the program for non-college-bound students, and the role of consumer mathematics in the math program. Teachers, counselors, and principals were surveyed on these issues. In addition, parents' answers to general questions about the APS math program often highlighted these issues.

Algebra. The majority of respondents expressed the opinion that algebra I should be taught first in the ninth grade. Seventy-four percent of the secondary math teachers, 40% of principals, and 57% of counselors expressed this opinion. The grade level selected by the second largest group was the tenth grade, chosen by 17% of math teachers, 16% of principals, and 28% of counselors. The third choice was to offer algebra in the eighth grade, selected by 5% of the teachers, 8% of the principals, and 7% of the counselors. Eleventh grade was not considered appropriate for beginning algebra.

Calculus. The role of calculus in the high school program is the focus of intense debate. Some educators believe that since a small number of high school graduates will require advanced mathematics in their careers, time is better spent studying the non-career math which will be useful in everyday life (Sonnabend, 1985). Some college level math professors oppose calculus in the high school and believe it should be offered only at the college level. Others see a place for calculus when offered as an Advanced Placement (AP) course (Kenelly, Henry, & Jones, 1985). Still others do not believe calculus is the appropriate starting point for the study of applied math or fields that involve math because of the impact of computers on mathematics (Ralston, 1985).

Teachers, principals, and counselors in APS were asked for what percent of the high school population they felt calculus was appropriate. Thirty-three percent of of math teachers felt calculus was appropriate for 2% or less of the population, while 23% selected between 3% and 5%. Fifteen percent of math teachers selected 6-10%, 8% selected 11-25%, and 3% selected 26% or more. About half of the principals did not express an opinion on this item. But those principals who did respond believed calculus was appropriate for many more students than did teachers. Counselors also were more inclined than secondary teachers to view calculus as appropriate for a greater percentage of high school students. See Table 6.

Table 6
Appropriateness of Calculus
for High School Students

	Teachers	Principals	Counselors
Less than 2%	33	5	19
3-5%	23	14	26
6-10%	15	15	21
11-25%	8	17	16
26% or more	3	3	2

In APS approximately 1.1% of the high school students take calculus. The discrepancy between the opinions of APS personnel and the actual percentages of students taking calculus, as well as nationwide controversy about calculus, suggest that this area needs further clarification.

Sequencing of Algebra and Geometry. The most preferred sequencing of high school courses is algebra I, geometry, and algebra II, while the sequence algebra I, algebra II, and geometry is less popular (Manhard, 1985). In APS

the greatest percentage of math teachers (48%) preferred the latter sequence with algebra II directly following algebra I. Those counselors and principals who expressed an opinion tended to prefer the algebra I, geometry, algebra II sequence.

Non-College Preparatory Program. Survey respondents were asked if they believed algebra and geometry should be integrated into the program for non-college-bound students. The majority of math teachers (67%), principals (57%), and counselors (59%) were in favor of this addition. This is consistent with Usiskin's (1983) recommendation that programs for non-college bound students include applied algebra and geometry.

Consumer Mathematics. Although parents were not asked specifically about consumer mathematics (math that emphasizes the skills needed for daily living), many parents expressed the opinion that this should be part of the math preparation of all students. In contrast to the opinions of school personnel, some parents questioned the importance of algebra, geometry, and calculus for those students who weren't planning careers in mathematical fields. They suggested that these students, as well as college-bound students, needed more practical math skills.

Teaching Strategies

In this section two major topics are addressed. These are the amount of time devoted to mathematics instruction, and the type and frequency of use of various teaching strategies in mathematics classes with a particular focus on math homework.

Time Spent on Math Instruction

Research in recent years has shown a significant relationship between the amount of time spent on learning and student achievement (Walberg, 1984). In response, the Association for Supervision and Curriculum Development (ASCD) undertook a study to determine how much time is allocated at the elementary school level to each content area. Their findings based on 1500 principals' reports of time allocations at the fourth grade indicated that the largest block of time is allocated to reading and language arts with an average of 100 minutes per day. The second largest block of time is devoted to mathematics, with an average of 52 minutes per day (Cawelti & Adkisson, 1985). Goodlad (1984) reported a similar finding, that an average of 54 minutes a day is spent on mathematics. Substantial variation exists among schools and individual classrooms depending on teachers' competence in or preference for particular subjects. In the ASCD study, 7.2% of principals reported an average of three or less hours per week, 36.6% reported an average of between three and one-fourth to four hours, 49.7% reported an average of between four and one-fourth to five hours, and 6.5% reported an average of more than five hours of math instruction a week.

In APS, elementary teachers were asked to indicate how much time they spent on math instruction and activities each day. As in the ASCD study, a great deal of variation was evident. Seven percent of the teachers reported spending 20 minutes or less each day, 13% spent between 20 and 30 minutes, 6% spent between 30 and 40 minutes, 26% spent 45 minutes, and 8% spent a full hour a day on mathematics. The overall average in APS was 46 minutes a day. Therefore, 20% of APS teachers allocate three hours or less to math each week, 26% allocate between three and four hours, 35% allocate between four and five hours a week, and none of the elementary teachers surveyed allocate more than five hours a week to math. It should be noted in interpreting these results that principals' reports of time allocations in the ASCD study may not correspond to the amount of time teachers actually engage in mathematics instruction.

At the secondary level the amount of time devoted to mathematics is limited by the length of the class period. In APS this varies from 42 to 50 minutes.

Frequency of Use of Teaching Strategies

A study sponsored by the National Science Foundation indicated that lecture and discussion are the predominant teaching strategies used in mathematics classes regardless of grade level (Weiss, 1977). Individual assignments, chalkboard work, and tests were found to be a fairly routine part of math instruction, while use of alternative activities such as library work, student projects, field trips, and guest speakers were not common. APS teachers were asked to indicate how frequently they used 17 different strategies in teaching math. Textbook assignments were the most frequently used strategy by both

elementary and secondary teachers, which is consistent with the finding of the NSF study that the textbook plays a central role in mathematics classes (Weiss, 1977). The role of the textbook will be discussed in more detail in the section on materials. Another frequently used strategy at the secondary level was homework, with 91% of secondary teachers reporting that they gave homework assignments almost every day.

Drill and practice were also quite common across grade levels. While lecture was one of the most frequently used strategies at the secondary level, it was one of the least frequently used strategies at the elementary level. Elementary math instruction incorporated a broader range of strategies than did secondary math instruction. Strategies used frequently at the elementary level, besides textbook assignments and drill and practice, included discussion, chalkboard work, small-group learning, peer tutoring, homework, and individual conferences. These strategies were used at least once a week or just about daily by more than 50% of the elementary teachers. Many other strategies were used on an occasional basis by elementary teachers. In contrast, at the secondary level, six strategies dominated math instruction. These were homework, textbook assignments, lecture, chalkboard work, drill and practice, and discussion, which were used at least once a week or just about daily by 85-90% of the teachers. Two other strategies, peer tutoring and individual conferences, were used by about half of the secondary math teachers. Use of other teaching strategies was rare in secondary classrooms.

The types of strategies used least frequently in mathematics classes at both levels were almost identical and included: guest speakers, field trips, library work, team teaching, and student projects. Interest centers were not used by most secondary math teachers, but elementary teachers reported all levels of use of interest centers from "never" to "just about daily." This may be a reflection of whether teachers use interest centers for any instructional area in their classroom rather than a reflection of the appropriateness of interest centers as a particular strategy for teaching math.

Homework. Because of an intense focus in recent years on the role of homework in the instructional program, several questions about homework were asked of teachers, students, and parents. Ninety-one percent of secondary math teachers and 27% of elementary teachers reported that they gave homework almost every day. When students were asked about homework assignments, 59% of high school students and 34% of middle school students said they had homework every night, while 33% of high school students and 54% of middle school students reported they had homework assignments two to four times a week.

Students at all levels were asked who helps them with their homework. The largest number of elementary students reported that their mother helped them, followed by their father and their teacher. At the middle school level, equal numbers of students indicated that their father and mother helped them, followed by a category of other, and the math teacher. High school students primarily received help from other students, followed by the math teacher and their father. High school students also were asked how often they needed help with their homework. Forty percent of the students said they needed help sometimes, 20% said not very often, 19% said almost always. High school students for the most part spent an hour or less a day on math homework. Twelve percent spent less than 15 minutes, 32% spent between 15 minutes and a half hour, and 41% spent between one-half hour and one hour. Thirteen percent of the students surveyed indicated they spent between one and two hours a night on math homework.

Parents were asked if they felt comfortable helping their children with their mathematics homework assignments. More than a third (37%) felt quite comfortable helping their children, while 36% felt somewhat comfortable, and 21% did not feel comfortable. However, in response to open-ended questions, many parents indicated that they did not understand the way math was taught today and were interested in classes or workshops on math for parents. In addition, some parents commented that the explanations in textbooks frequently were inadequate and they were, therefore, unable to help their children with assignments.

"My whole life passed before my eyes except the part where I learned the multiplication tables."

Math Materials

The materials used to teach mathematics are of special interest today with the increasing availability of technological aids such as computers and calculators. It is important to know which materials are actually being used in mathematics classrooms, how frequently they are being used, and how adequate teachers find the available materials. Each of these topics will be discussed in this section, with a special focus on the mathematics textbook, manipulatives, calculators, and computers.

Frequency of Use of Materials

The most frequently used material in mathematics classes is the textbook. Seventy-one percent of elementary teachers and 93% of secondary teachers in APS use the math textbook just about daily. At the elementary level, the next most frequently used material is manipulatives, with 44% of teachers using manipulatives almost daily and 24% using them at least once a week. In addition, elementary teachers report that they use duplicated worksheets frequently-67% of teachers use these at least once a week or just about daily. Although most elementary teachers do not use computers daily, 10% of these teachers report daily use and 34% report that they use computers at least once a week to teach math. Other materials such as supplemental books, audiovisual aids, and calculators are used infrequently at the elementary level.

At the secondary level, not only do 93% of math teachers use the textbook almost everyday, practically no other material is used in the teaching of mathematics on a daily basis. Supplemental books are used by 40% of the teachers and duplicated worksheets are used by 54% of the teachers at least once a week. Calculators are used more frequently than at the elementary level; 36% of secondary teachers use them at least once a week or daily compared to 11% of elementary teachers. However, computers are used less frequently-nearly half (47%) of the teachers never use computers and 34% use them less than once a month compared to 27% of elementary teachers reporting that they never use computers and 9% that they use them less than once a month. Manipulatives are used infrequently in secondary mathematics classes-65% of the teachers never use them or use them less than once a month. Like elementary teachers, audiovisual aids are seldom used by secondary math teachers. See Table 7.

Table 7
Percent of Teachers Reporting
Daily and Weekly Use of Math Materials

	DAILY		WEEKLY	
	Elementary	Secondary	Elementary	Secondary
Textbook	71	93	9	6
Manipulatives	44	2	24	10
Duplicated Worksheets	30	11	37	54
Calculators	3	17	8	19
Computers	10	3	34	6
Supplemental Books	10	5	20	40
Audiovisual Aids	11	12	15	13

In summary, the most frequently used materials in elementary classrooms are the textbook, manipulatives, duplicated worksheets, and computers. At the secondary level practically no material other than the textbook is used everyday, but on a weekly basis secondary teachers use duplicated worksheets, supplemental books, and calculators. Teachers at all grade levels rarely use audiovisual aids.

Adequacy of Available Math Materials

Teachers were asked to rate the quantity and quality of materials available to them for use in their mathematics classes.

Quantity of Materials. Secondary math teachers had more definite opinions regarding the quantity of math materials. More than 80% of these teachers felt there were sufficient numbers of student textbooks and teachers' editions of textbooks. About half thought supplemental materials provided by the publishers were adequate in terms of quantity. However, more than half indicated that there weren't enough computers or calculators for classroom use, which may be why these materials are not used more frequently in teaching math. Interestingly, 39% of secondary math teachers also thought the quantity of manipulatives was inadequate.

Elementary teachers, for the most part, felt the quantity of student textbooks, teachers' editions, and manipulatives were adequate. Although more elementary teachers than secondary teachers reported that the number of available computers was adequate, 41% rated the quantity as inadequate. Principals shared opinions similar to teachers. Although they felt textbooks were adequate in terms of quantity, about half of the principals rated the number of computers, calculators, and manipulatives as inadequate. Counselors generally were not close enough to the classroom to be able to rate these items.

Quality of Materials. Overall, the majority of secondary teachers expressed the opinion that math materials were adequate in terms of quality. The largest percentage of teachers (79%) thought the teachers' editions of textbooks were of adequate quality, followed by student texts (74%), computers (64%), and supplemental materials (61%). Less than half of the teachers believed calculators (47%) and manipulatives (43%) were of adequate quality.

Again, elementary teachers were less likely than secondary teachers to express opinions about the quality of math materials, but of those who did rate the materials, opinions generally matched those of secondary teachers, with one exception. Seventy percent of elementary teachers rated manipulatives as adequate quality while only 43% of secondary teachers did so.

Principals opinions concurred with the opinions of teachers although a greater percentage indicated that texts, both teachers' editions and student texts, were of adequate quality, while fewer principals than teachers felt computers and publishers' supplemental materials were adequate in terms of quality. Most counselors did not express opinions regarding the quality of math materials.

Some parents expressed the opinion in written comments that their children's textbooks were not adequate in terms of explaining how to solve problems. They mentioned that explanations were not only inadequate for the students,

but also for parents when they attempted to help their children with their homework. It is not possible to ascertain whether this is due to inadequate texts or to insufficient knowledge of how mathematics is taught today. Nevertheless, it is an area of concern.

Respondents also were asked how appropriate district-provided math materials were for different groups of students. Teachers, principals, and counselors indicated that materials were somewhat appropriate for most groups. However, materials were perceived to be less appropriate for three groups-English as a Second Language (ESL) students, developmental/remedial students, and low achievers. Secondary teachers, principals, and counselors especially noted this. Math materials were rated comparatively more appropriate for females, minority students, and the mathematically talented.

Although school personnel basically found district-provided math materials to be adequate in terms of both quantity and quality, many were unaware of the procedures used by APS to select math materials. Fifty-four percent of elementary teachers, 42% of secondary teachers, and 77% of counselors were not familiar with materials selection procedures. In contrast only 12% of principals were not aware of how the school district selects math materials. This lack of familiarity could be addressed by providing teachers with a short memo about materials selection procedures.

The Mathematics Textbook

Research has shown that the textbook is not only central to instruction but frequently determines the math curriculum (Lindquist, 1984; Raizen & Jones, 1985). Stake and Easley (1978) in a set of case studies sponsored by the National Science Foundation found that 90% of science and math teachers use a textbook 90-95% of the time. A review of 20 years of research indicated that a single text is used in most classrooms and that many teachers use no instructional materials except the textbook and the chalkboard (Suydam & Osborne, 1977). This heavy reliance on textbooks is of concern since publishers of textbooks frequently are not knowledgeable in mathematics or pedagogy and because publishing as a business, is motivated to some extent by profit (Lindquist, 1984). In addition, Driscoll (1980) points out that the major math textbooks in use in the United States emphasize low level cognitive processes and computational skills and are based on "conventional wisdom" rather than research. In spite of these limitations, the textbook is perceived to be very important, especially by parents, in determining the content of the math program. In addition, in all mathematics classes in APS the textbook is the most frequently used teaching material while in secondary math classes practically no other material is used on a daily basis.

Since textbooks are relied upon so heavily in APS math classes, the selection of textbooks is extremely important. Every effort should be made to familiarize teachers with procedures for materials selection and textbook adoption and to encourage their active involvement. Also, in keeping with NCTM'S (1980) recommendation that problem solving be the focus of school mathematics in the 1980's, textbooks should be carefully evaluated to ensure that they stress higher level cognitive skills such as problem-solving and applications of mathematics. Finally, an increased emphasis should be placed on the integration of manipulatives, calculators, and computers (NCTM, 1980).

The Use of Manipulatives in Teaching Mathematics

Manipulatives are objects which appeal to several senses and which a student is able to touch, move, and handle (Driscoll, 1980). According to Piaget, children progress through several cognitive levels and even at the age of 12 deal well with symbols only if they are closely tied to perceptions (Driscoll, 1980).

Suydam and Higgins (1977) in their review of research found that in almost half of the studies, students who learned math with the help of manipulatives scored significantly higher on achievement tests than students whose instruction did not include the use of manipulatives. Another study found that manipulatives improved learning and retention, as well as the ability to transfer knowledge to problem-solving situations (Prigge, 1978). "The overriding concensus is that manipulatives can help children to understand and use mathematical concepts" (Driscoll, 1980, p.3). Yet half of K-6 teachers surveyed nationally reported that their students use manipulatives less than once a week or not at all (Fey, 1979). In APS the average is quite a bit higher, with 44% of the elementary teachers using manipulatives almost daily and 24% using them at least once a week. However, 63% of APS secondary math teachers never use manipulatives or use them less than once a month.

Issues involved in using manipulatives to teach mathematics include: the perception of the importance of using manipulatives at various grade levels, the adequacy of teacher training in incorporating manipulatives into the mathematics program, the availability of manipulative materials, and how manipulatives are currently being used.

Perceptions of Importance. Research indicates that even as children become more adept at using symbols, there is still a place for manipulatives in the intermediate elementary grades and into the secondary level (Driscoll, 1980). Perceptions regarding the importance of using manipulatives at different grade levels varied in APS, with teachers and principals expressing the opinion that manipulatives are very important at the primary (K-2) and intermediate (3-5) grade levels, somewhat important at the middle school level, and slightly less important at the high school level. Counselors saw manipulatives as somewhat important at all grade levels, while parents felt that it was somewhat to very important for their children to use concrete objects in learning math.

Elementary and middle school students were asked if they enjoyed working with real objects like cuisenaire rods or puzzles. Fifty-eight percent of the elementary and 45% of the middle school students responded affirmatively. In addition, 53% of high school students said that using a diagram or a real object helped them solve problems.

Availability of Manipulatives and Training. Regardless of perceptions of importance of using manipulatives, if materials are not available or teachers do not feel skilled in using them, they will not be used. Elementary and secondary teachers were asked how adequate their training has been in the use of manipulatives. Eighty-nine percent of elementary teachers and 62% of secondary math teachers rated their training as somewhat to very adequate. Although training for the most part was considered adequate, 34% of elementary teachers and 41% of secondary teachers indicated that an insufficient number of manipulative materials were available to them. Some elementary teachers

reported that the manipulative kits they "inherited" as new teachers were missing parts or that the previous teacher had taken the manipulative kit to a new location leaving them with none. As of this date additional manipulative kits have been supplied to elementary teachers who attended inservice sessions on their use.

Use of Manipulatives. Research indicates that manipulatives are useful in developing concepts and skills as well as in providing remedial instruction (Ashlock, 1976). Elementary and secondary teachers in APS were asked how they used manipulatives in teaching mathematics and the survey results show a variety of uses. More than 90% of the responding elementary teachers indicated that they use manipulatives to introduce, reinforce, and remediate concepts and skills. Seventy-nine percent said they used manipulatives to expand concepts and skills, 78% to create problem-solving situations, and 71% to explore concepts and skills. Of those secondary math teachers who use manipulatives the greatest percentage (71%) do so to introduce concepts and skills, followed by reinforcing and remediating (51% each), exploring (46%), expanding (44%), and creating problem-solving situations (36%).

From these results it appears that there is a need for more manipulatives as well as some training for teachers, especially upper level teachers, on how manipulatives can be incorporated into the teaching of mathematics.

Calculator Use in Mathematics Instruction

The NCTM (1980) and the Conference Board of the Mathematical Sciences (1983) have recommended that mathematics programs take full advantage of the use of calculators. The attitude of most mathematics teachers toward calculator use has changed from caution to enthusiasm although elementary teachers still are generally not as favorable as secondary teachers (Driscoll, 1982). Driscoll (1982) reported that over 100 research studies have compared groups of students who have used calculators with groups of students who have not used calculators in mathematics. Scarcely any of the studies showed an advantage in math achievement for the noncalculator groups and most studies indicated either no significant differences or an advantage for the students who used calculators (Suydam, 1979). Therefore it was deemed important to discover the attitudes toward calculator use of APS teachers, principals, parents, and students.

Attitudes Toward Calculator Use. Teachers and principals at all levels saw calculator use as correspondingly more appropriate as students matured. Calculator use was considered by school personnel to be less appropriate for primary students and more appropriate for high school students, although elementary teachers and principals were more favorable towards the use of calculators at all levels of elementary and middle school than were secondary teachers. Parents, as a group, thought the use of calculators in math classes was important, 38% rated their use as somewhat important, and 25% rated their use as very important.

The majority of students at all levels did not see calculators as a substitute for learning mathematics. Seventy-three percent of elementary students and 79% of middle school students did not agree that using a calculator made it unnecessary to learn math. Sixty-four percent of high school students disagreed that frequent calculator use made them forget how to do arithmetic. In addition, 69% of elementary and 68% of middle school students felt that using calculators made math easy.

Uses of Calculators. Several studies have shown that perceived uses of calculators affect teacher attitudes. Wyatt (1979) found that teachers who had never used calculators in the classroom were aware of only two uses: computation and checking answers. Yet the benefits of calculator use encompass a broad range of applications such as: teaching problem solving, estimation, and reasonableness; exploring, discovering, and developing concepts across the entire math curriculum; evaluating the depth of students' understanding of mathematics; and diagnosing misconceptions and difficulties (Driscoll, 1980,1982; CBMS, 1983; NCTM, 1980).

Elementary teachers, who as a group use calculators infrequently in mathematics classes, indicated that when they have students use calculators, they are used most frequently for special activities, checking answers, exploring problems, and in helping students whose computation skills interfere with problem solving. Less than 25% of the elementary teachers reported occasional or frequent use of calculators for computing answers, learning basic facts, developing math ideas or concepts, or introducing new concepts.

A greater percentage of secondary math teachers felt the number of calculators available to them was insufficient, yet reported much more frequent use of calculators in the classroom, perhaps because secondary students are more likely to own a calculator. Secondary teachers' three most frequent uses of calculators were to help students whose computation skills interfere with problem solving, for special activities, or to check answers. They also occasionally used calculators to help students compute answers and explore math. Like elementary teachers, secondary teachers rarely used calculators to develop math ideas and concepts, to introduce new concepts, or to help students learn basic facts.

About a third of elementary teachers permit calculator use on classroom assignments and homework, though only 10% permit their use on tests. In contrast, more than half of the secondary math teachers allow students to use calculators on homework and classroom assignments, while a third allow calculator use on tests. High school students also were asked how frequently they use calculators in math classes and on homework. Fifty-two percent of students said they almost never use calculators in their math class, while 34% said they use them occasionally. About a third of the students reported that they use calculators several times a week or on a daily basis in their math classes. Twenty-five percent of middle school students and 18% of elementary students said they sometimes use a calculator at school. These results reflect teachers' beliefs that calculators are more appropriate in the upper grades. More students at all levels use calculators at home than they do at school. A third of high school students indicated that they almost never use calculators at home, while 48% said they occasionally did. A small percentage (17%) indicated they almost always use calculators for their homework. Almost half (49%) of middle school students and 23% of elementary students use a calculator to do their math homework.

Driscoll (1982) believes that the calculator is in a peculiar situation. NAEP data show that 80% of 13-year olds and 85% of 17-year olds either own a calculator or have one available for their use, but calculators are relatively invisible in classrooms. Introducing calculators into the curriculum, he states, requires plans for purchase, maintenance, teacher training, etc. (Driscoll, 1980). Reys et al. (1980) found that a large percentage of secondary math teachers wanted training in classroom uses of calculators but

scarcely any had received training. The results of the surveys in APS on calculator use indicate that students have calculators available and enjoy using them. In addition, they indicate that many teachers are receptive to their use in certain areas. However, some of the beneficial uses of calculators, for example in developing math concepts, have yet to become widespread, indicating a need for further research and inservice on the applications of calculators in mathematics.

Computer Use in Mathematics Classes

No one seems to argue with the notion that teaching students to use computers is important. Locally, 58% of parents think it is very important for their children to use computers in math classes, while 29% think it is somewhat important. Computers are rapidly becoming standard equipment in schools across the nation. NAEP (1983) data show there has been a significant increase in computer literacy courses and enrichment activities or in the high school semester or year-long courses that are electives. The National Council of Teachers of Mathematics (1985) points out that math curricula, instructional methods, and teacher preparation are being completely reshaped by computers, calculators, and other electronic information technology. The NCTM (1985) believes that, "The major influence of technology on mathematics education is its potential to shift the focus of instruction from an emphasis on manipulative skills to an emphasis on developing concepts, relationships, structures, and problem-solving skills" (p.1). The Conference Board of Mathematical Sciences (1983) also recommends that calculators and computers be introduced into the mathematics classroom as early as possible and used to enhance the understanding of arithmetic, geometry, and problem-solving skills.

Classroom teachers in APS do not use computers very often in teaching math, although elementary teachers use them much more frequently than high school teachers. A third of elementary teachers use computers at least once a week to teach math and 10% use them almost on a daily basis. Secondary teachers, on the other hand, practically never use computers in their math classes. Only 6% use them once a week, while 3% use them almost daily. Eighty-one percent of secondary teachers, as opposed to 36% of elementary teachers reported that they use computers less than once a month or never use them.

This is substantiated by students' responses to questions about computer use. Seventy-seven percent of elementary and 60% of middle school students said they liked to use the computer for math activities such as LOGO or Gertrude's Puzzle. However, only 29% of high school students said they enjoyed this, while 49% expressed no opinion, probably indicating that they had not had this experience. In fact, 86% of the high school students who were polled said they never use a computer in math classes. Nine percent reported use of a computer once a semester and 4% reported monthly use.

Clearly, at this point in time computers are underutilized in math classes. It is not known whether this is due to insufficient numbers of computers or lack of knowledge regarding the special applications of computers in math classes, or a combination of the two. Fifty-six percent of secondary teachers and 41% of elementary teachers believe that an inadequate quantity of computers is available. However, it is equally likely that, as with calculators, special training for teachers on particular applications of computers in math will greatly facilitate their use.

Teacher Preparation

During the 1970's there was a decline of 77% in the number of mathematics teachers being prepared. In 1981, 43 states reported a shortage of mathematics teachers and according to some estimates nearly half of beginning mathematics teachers are not fully certified to teach math (Hurd, 1982). In addition, elementary teachers are expected to be equally well prepared in all subject areas, yet typically have had only one course in math methods.

Adequacy of Math Preparation

In order to ascertain APS teachers' perceptions of the adequacy of their math preparation, teachers were asked if they'd ever had to teach or were currently teaching a math unit or course for which they felt unqualified. Twelve percent of elementary and 72% of secondary teachers said this had happened once. Twenty-eight percent of elementary and 13% of secondary teachers indicated this had occurred a few times, and 12% of secondary math teachers said this was a frequent occurrence. Seventeen percent of principals also believed that there are math teachers in their own schools who are currently teaching or have taught math courses for which they lacked qualifications.

Teachers indicated that what best prepared them to teach math was their actual experience. This was true of both elementary and secondary teachers. However, the two groups diverged at this point. Elementary teachers indicated that the next best preparation was provided by: workshops and inservice, the methods and materials courses they had taken, student teaching, math content courses, and their own pre-college background in mathematics. Secondary teachers, on the other hand, found the math content courses they had taken to be the next best preparation. This was followed by student teaching, their pre-college background, and workshops and inservices. Secondary teachers found methods and materials courses to be the least helpful in preparing them to teach math. These results are not surprising when you consider the nature of the mathematics taught by elementary and secondary teachers and the typical teacher preparation of these two groups. But they do suggest that the best ways to prepare teachers to teach math may be different for elementary and secondary teachers. The NCTM (1980) has made several recommendations regarding teacher certification in math, but they are beyond the scope of this report.

Teacher Training and Inservice

Teachers also were asked what kinds of topics should receive the most emphasis in teacher education programs. Again, their responses varied according to the grade level they taught. Elementary teachers felt the most important topic was math methods, followed by math content, sensitivity to student needs, classroom management, diagnostic and remediation strategies, developing math materials, and student evaluation techniques. Secondary teachers thought math content and classroom management were most important, followed by: math methods, sensitivity to student needs, diagnostic and remediation strategies, developing math materials, and student evaluation techniques.

Once they are in the classroom, elementary and secondary teachers indicated that other teachers were their best sources of information on new math content, strategies, and materials. In addition, elementary teachers found

school-based and district-wide inservice and the district math specialists to be helpful. They did not find journals and professional publications, meetings of or membership in professional organizations, the principal, or publishers to be particularly helpful in providing information on the teaching of mathematics. Secondary teachers, in contrast, found publications and meetings of professional organizations to be fairly useful. On the other hand, they did not think that inservices or the district math specialists were as helpful as did elementary teachers. But like elementary teachers, they too, did not consider publishers or principals able to provide them with information on mathematics.

While between 25-36% of secondary math teachers belong to professional math organizations, only 2-3% of elementary teachers hold such membership. Eight percent of both groups of teachers are members of more general educational organizations such as ASCD and Phi Delta Kappa (PDK). Naturally, since elementary teachers teach all areas, their membership in professional organizations will reflect special interests, but conferences or meetings of these organizations might prove to be beneficial. In order for this to occur, however, the NCTM (1980) beleives that administrators must facilitate involvement of teachers, without penalty, in conferences and professional work.

Both groups of teachers subscribe to math publications or read them on a regular basis. Twelve percent of elementary teachers and 39% of secondary teachers have subscriptions and 33% of elementary and 60% of secondary math teachers read one or two math journals at least once a month.

These results suggest that facilitating interaction among classroom teachers may be one of the best ways to improve the teaching of mathematics. Indeed, this is one of the recommendations by NCTM (1980) to raise the professional standards of teachers. In addition, an effort should be made to provide appropriate inservice for teachers at all levels and especially to discover what kinds of inservice secondary teachers would find helpful. Finally, teachers should be encouraged to actively participate in professional organizations, conferences, and the reading of mathematics journals and publications.

"Excuse me, Mr. Claybill, but if you've been teaching math for 35 of your 40 years, doesn't that mean you started when you were 5 years old?"

Effectiveness

School personnel and parents were asked to indicate how the effectiveness of the APS mathematics program should be determined. All groups agreed that teacher/counselor opinions were the most important in determining effectiveness. This was followed by mastery of the mathematics key competencies in grades K-8, and the results of teacher-made tests. Standardized test scores, parental opinion, and administrator opinion were rated as somewhat important in determining effectiveness. Slightly less important, but still considered somewhat important were student opinion and community opinion.

These results indicate that the opinions of professional educators, as well as the achievement of students on a variety of tests are considered important indicators of an effective mathematics program. In keeping with this finding, this section will include information from several sources to provide an overall picture of the effectiveness of the APS mathematics program. First, the opinions of teachers, principals, counselors, parents, students, and community members about the effectiveness of the program will be presented. Second, student achievement data on the APS Key Competencies and the Comprehensive Tests of Basic Skills will be presented. Third, student attitudes toward mathematics will be discussed, as student attitude is an important indicator of an effective math program. Finally, obstacles to effectiveness will be discussed.

Opinions of Effectiveness

Teachers, principals, counselors, and parents believed that the APS mathematics program is the most effective in preparing students for further education, teaching basic mathematical knowledge, and preparing students to use math in daily living. In addition, they also believed APS to be somewhat effective in teaching mathematical thinking and problem solving and preparing students for jobs. These groups indicated that they thought the math program was slightly less effective in teaching students to enjoy math for its own sake and enabling students to enjoy math in their leisure time.

There was a correlation between the ratings of the importance of the goals of mathematics and the ratings of the effectiveness in achieving them. The three goals that were considered most important: teaching basic math knowledge, preparing students for further education, and enabling students to use math in daily living, were also the goals that were rated as having been most effectively achieved. The goals that were considered less important were not believed to be accomplished as effectively.

Survey respondents also were asked how well they thought the mathematics program at various levels prepared students for the next higher level. For the most part, the majority of all groups surveyed expressed the opinion that the mathematics program at each level was somewhat effective in preparing students for the next level. A smaller percentage, between nine and 20 percent rated the program as very effective in preparing students for the next higher level of mathematics. Students also thought they had been well prepared in mathematics. Ninety-one percent of elementary students thought they had learned a lot of math in school. Sixty-nine percent of middle school students believed that the math they learned in elementary school prepared them well for middle school math classes and 51% of high school students felt

their elementary and middle school math classes were good preparation for high school mathematics courses.

Employers in the Albuquerque area were interviewed to determine how well prepared they thought APS graduates were for jobs in their companies. Two of the seven employers reported that they never hire graduates immediately after high school, while five of them occasionally do so. For the most part, high school graduates were hired for low level, non-technical positions and sometimes were thought to lack basic math skills in use of fractions and decimals, estimation, and assessing "reasonableness" of answers. Three of the employers who do hire APS high school graduates remarked that the students they hired had good clerical skills, business math skills, and data processing skills.

Student Attitudes Toward Mathematics

Elementary, middle, and high school students completed surveys which measured attitudes toward mathematics. Some of the items were similar or identical to items included in the National Assessment of Educational Progress (1983).

An interesting profile emerged. Elementary students generally held favorable attitudes towards mathematics. The majority enjoyed tackling difficult problems and trying out their own methods for solving problems. They thought math was interesting, felt confident of their ability, and thought they would enjoy using mathematics in their future careers. However, by middle school these attitudes had changed dramatically. Slightly more than half of middle school students still thought math was interesting, but fewer felt confident of their ability, and the majority no longer enjoyed working difficult math problems, especially word problems. Less than half now thought they'd like to use math in their career or vocation. By high school, the attitudes changed only slightly, although in almost all instances these responses were less positive than for middle school students. Half of the high school students thought word problems were difficult, while at the elementary level 53% of students liked doing word problems. Fewer high school students liked to figure out problems their own way and tended to see math in a more rigid way, as mostly memorizing facts. Only 40% of secondary students thought they'd enjoy using math in their future job.

In spite of this negative change in attitude from elementary to high school, student attitudes in some areas are encouraging. More than half of the students at all levels felt confident of their ability in math. Most of the students reported that they felt good when they could solve a math problem by themselves. Most students believed that girls and boys are equally adept at mathematics. More than ninety percent of the students thought their parents wanted them to do well in math, and the majority of the students, themselves, thought math was useful outside of school. Between 40% and 62% of students believed they would enjoy using mathematics in their career or vocation.

Students were aware that there is more to mathematics than merely getting the correct answer. Although half of elementary students think getting the right answer is the most important thing, this drops to a third of the students at middle school. More than 80% of high school students believe that understanding how to do a problem and knowing why an answer is correct is as important as getting the right answer. This response is comparable to the responses of 13 and 17 year olds in the NAEP (1983). In contrast to this

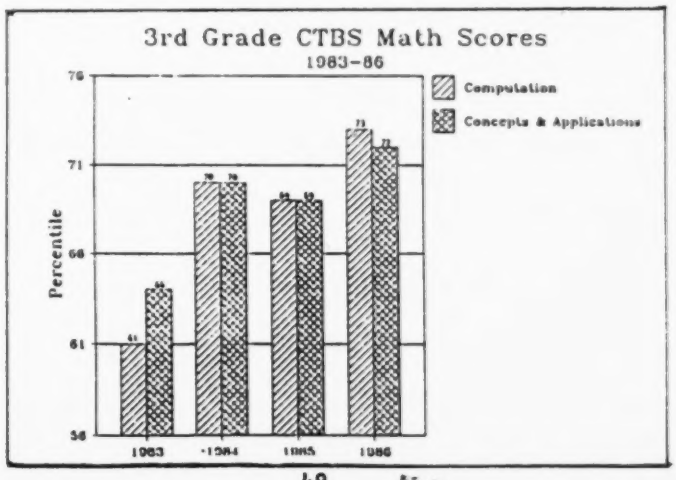
belief in the importance of understanding mathematics, between 40% and 53% of APS students, comparable to the NAEP results, believe math is mostly memorizing facts. Sixty-eight percent of high school students believe there is always a rule to follow in solving problems. Although this is a large number, it is smaller than the 88% of students who reported their agreement with this statement on the NAEP (1983). Also, like the national sample, half of APS high school students believe math is made up of unrelated topics and about a quarter of the students disagree that mathematicians work primarily with symbols rather than ideas. Fey (1979) does not find these attitudes surprising since most of students' experience with math has involved memorizing and has not focused on new discoveries in math or interrelationships among mathematical topics.

Student Achievement in Mathematics

Each spring, the Comprehensive Tests of Basic Skills is administered to third, fifth, and eighth graders in APS. In the area of mathematics, the test measures students' skills in math computation as well as math concepts and applications. In addition, twice a year in September and in May, students in grades one through eight are tested using the district-developed Key Competencies in Mathematics tests. The tests focus on computation, problem solving, and math concepts. At the high school level, students in grades nine and ten participate, at the school's option, in the testing of algebra skills using the Scott, Foresman High School Subject Test. Each of these three sources of data on student achievement will be discussed in this section.

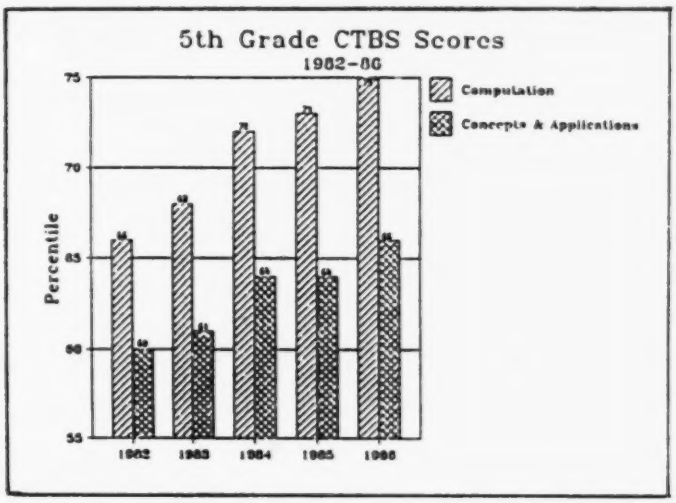
CTBS. At the third grade, data from 1983-1986 are available. Third graders were not required by the State to be tested prior to 1983. At the fifth and eighth grade levels, five years of data are available, between 1982 and 1986. The new form of the CTBS, form U, was first used in 1982 and cannot be compared to test results of previous years. Although each year's data represent a different group of students, trends over several years' time can be seen as a general indication of program effectiveness. The skills of third grade students during this time period are substantially above the national average, and have showed steady growth, from the 61st percentile in computation in 1983 to the 73rd percentile in 1986, and in concepts and applications, from the 64th percentile to the 72nd percentile. It can also be seen that third graders' skills are fairly even in both areas of computation and application of mathematical knowledge. See Figure 1.

FIGURE 1



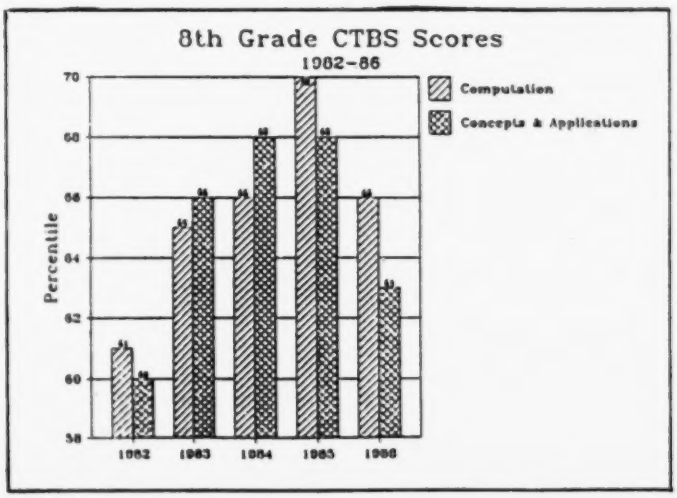
Fifth graders' scores also are substantially above the national average and have shown steady growth, however these students excel in computation skills and perform at a slightly lower level in the area of concepts and applications. In 1982, the district average for fifth graders was the 66th percentile in computation and the 60th percentile in concepts and applications. In 1986, fifth graders were at the 75th percentile in computation and the 66th percentile in concepts and applications. See Figure 2.

FIGURE 2



Eighth graders' skills, like third graders, are more even in both areas of math as measured by the CTBS. Although there was a drop in the average percentile from 1985 to 1986, eighth grade students' scores increased between 1982 and 1986. In 1982 the average score for eighth graders in computation was the 61st percentile and in 1986, the 66th percentile. In concepts and applications, the average score in 1982 was the 60th percentile and in 1986, the 63rd percentile. See Figure 3.

FIGURE 3



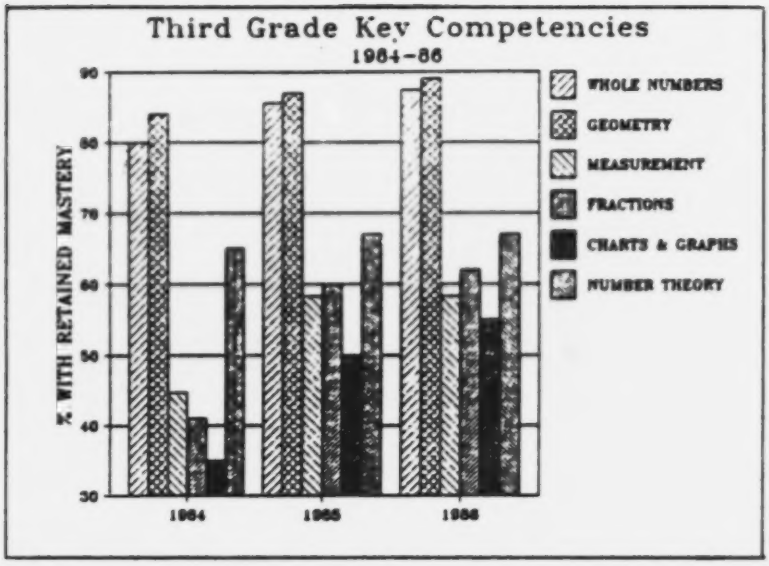
Key Competencies in Mathematics

The Key Competencies in Mathematics were developed in 1978, by Hilde Howden, the District Curriculum Coordinator for mathematics and the Math Advisory Council, composed at that time of teachers representing the elementary, middle, and high school levels. The competencies were based on a thorough review of textbooks and practices in other school districts throughout the country as well as the recommendations of professional math organizations such as the NCTM and the National Supervisors of Mathematics. The APS Key Competencies reflect the mathematical knowledge each child is expected to attain in grades kindergarten through eight. After the Key Competencies were piloted, a few minor revisions were made and grade level tests reflecting the Key Competencies were developed. In 1983, computer scannable forms were developed to record the results of the Key Competencies tests and district-wide results were tabulated for a few pilot schools. Prior to 1983, the primary focus was on the individual student; copies of the results were placed in the cummulative records so that the next year's teacher could use them in diagnosing areas of strength and weakness in mathematics. Although this focus has been retained, the test results are now also used to provide a district-wide picture of program effectiveness.

The Key Competencies tests and the CTBS are highly correlated and for the most part measure similar skills. The APS curriculum is based on the Key Competencies, while the CTBS matches fairly well what is taught. Results of the Key Competencies testing will be presented for third, fifth, and eighth grade students for 1984-1986. The results should be interpreted with caution for four reasons. First, the primary use of the Key Competencies is to diagnose individual student performance, not to assess program effectiveness. Second, there is a certain amount of subjectivity in rating students because some items are administered orally. Third, the Key Competencies do not receive the focus and attention in the schools that the CTBS does, and therefore, the administration might not be taken as seriously. Finally, it is impossible to know if students are not mastering certain areas due to the difficulty of the areas or due to the topics not receiving much attention.

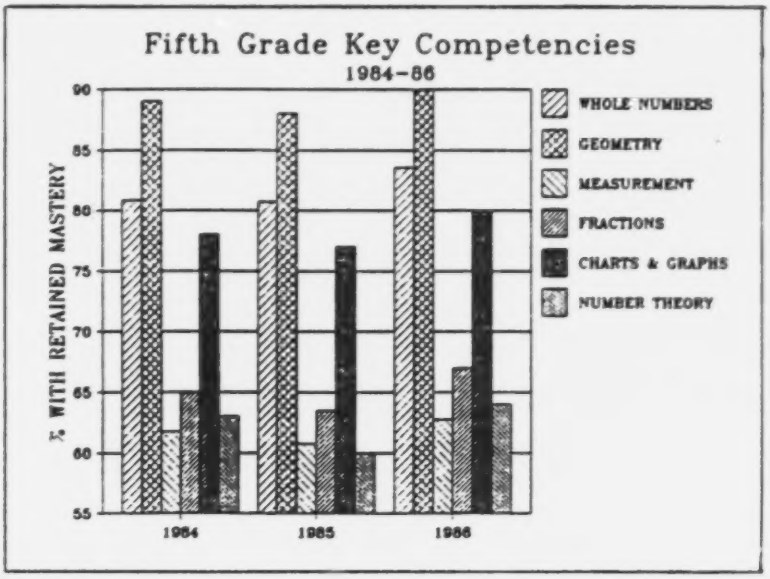
Between 1984 and 1986, more than 84% of third grade students attained mastery of geometry and whole numbers. Sixty-five percent or more mastered number theory each of these three years. Although less than fifty percent of the students mastered measurement, fractions, and charts and graphs in 1984, this percentage increased substantially in the following two years. See Figure 4.

FIGURE 4



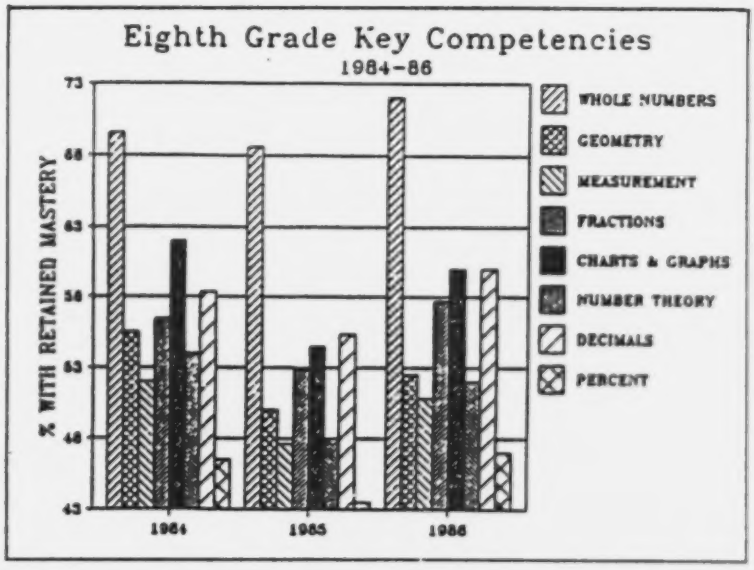
A similar pattern is evident with fifth grade students. The two areas mastered by the highest percentage of students are geometry and whole numbers. In addition, more than 7% of the fifth graders mastered charts and graphs and more than 60% of the students all three years mastered fractions, number theory, and measurement. See Figure 5.

FIGURE 5



At the eighth grade, the highest percentage of students, more than 69%, showed mastery of whole numbers. A substantial drop was noted between this category and the strands of mathematics mastered by the next highest percentages of students. These were charts and graphs, fractions, and decimals. Students in the eighth grade had more difficulty with the strands of geometry, number theory, measurement, and percent than with the other areas of study. See Figure 6.

FIGURE 6



In summary, the majority of students in grades three, five, and eight master whole numbers, geometry, charts and graphs, fractions, and decimals, while comparatively fewer master measurement and number theory. These results suggest that additional instructional time should be spent in addressing these topics. Also, in order for the testing to be less of a burden for teachers some aspects of the administration might be changed. Many of the optional comments provided by middle school teachers dealt with the Key Competencies. They indicated that hand scoring and filling in the bubbles on the computer scannable record sheets for all of the students in their classes was extremely time consuming. A few teachers questioned the value of the testing because they were unaware that the results were used in any meaningful way by the school district. Some teachers were of the opinion that the results were not easily accessible and, therefore, not likely to be used for instructional purposes.

Efforts have been made to address some of these issues. A pilot is currently being conducted to determine if both the fall and spring administration of the key competencies is necessary at the elementary level. At the middle school level, the fall testing was dropped after a similar study in 1981 showed a high correlation between the spring results and the fall results of the following year. In addition, the math department has worked with Data Services to improve the accessability and timeliness of the test results. Some middle school teachers apparently are unaware that Key Competencies results are made available to the following year's math teacher. Other teachers may be unaware of the best approaches to utilizing the test information they receive. Perhaps a solution might be to offer to math

teachers workshops, which would address ideas for the diagnostic and prescriptive use of Key Competencies results, much in the same way that CTBS workshops are offered. Pilots are also planned to shorten the number of days allowed for testing, to have students score their own tests, or to have students answer the test items on school computers so they can be machine scored. All of these efforts will continue to improve the ease of administration of the Key Competencies tests, as well as to improve the use of the tests for instructional purposes.

Scott, Foresman Algebra Test. During the 1982-83 school year, the Superintendent decided to discontinue testing with the CTBS at the 11th grade after the New Mexico State Department of Education made testing at this level optional. In its place, the Scott, Foresman High School Subject Tests were piloted in the spring of 1983 and field tested in the spring of 1984. Participation in the high school testing program is optional.

In 1985, nine schools used the algebra test and in 1986, five schools used this test. Since algebra is offered as a ninth grade course in APS high schools, most of the students taking the test were freshmen. However, some of the students in the sample were tenth graders because occasionally sophomores take algebra following a basic math course in ninth grade. Results from the algebra testing must be interpreted cautiously since they do not reflect the same population of students, grade levels, or schools each year and because teachers indicated that there is not a complete match between the algebra course content in APS and the content of the test.

The algebra test consists of 40 items assessing seven content areas: polynomials, exponents and expressions, definitions and theories, linear equations and inequalities, systems and coordinates, radicals and quadratics, and word problems. In 1985, students answered 41% of the items correctly, while in 1986, the average percent correct was 46. See Table 8.

Table 8

Scott, Foresman Algebra Test Results Summary
1985-86

Grade		1985	1986
9	Number of Students	721	1370
	Percent Correct	46	48
10	Number of Students	2261	463
	Percent Correct	40	40
11	Number of Students	77	
	Percent Correct	35	
District	Number of Schools	9	5
	Number of Students	3059	1833
	Percent Correct	41	46

When expressed in percentiles, the average score in 1985 was the 38th percentile, while in 1986, the average score was the 46th percentile. Analyzing the results separately by grade level shows that ninth grade students score substantially higher than tenth or eleventh graders. The discrepancy in scores across grade levels might be attributed to the ninth graders being more motivated because they are attempting this course earlier in their high school course of study. Furthermore, ninth grade students who elect algebra are likely to feel confident with their math backgrounds and may plan to take other more advanced math courses during high school. The eleventh graders, on the other hand, may be less motivated because they may be taking the course late due to a change in post high school plans or failure in the course earlier in high school.

In 1985, the ninth graders scored at the 45th percentile, the tenth graders at the 35th percentile, and the eleventh graders at the 26th percentile. In 1986, ninth graders scored at the 48th percentile and the tenth graders scored at the 40th percentile. No eleventh graders were tested in 1986. See Table 9.

Table 9

Scott, Foresman Algebra Test Results in Percentiles
1985-86

Grade	1985	1986
9	45	48
10	35	40
11	26	--
District	38	46

Obstacles to Effectiveness

Teachers in APS were asked what kinds of obstacles they faced in teaching mathematics to their students. Although the concerns of elementary and secondary teachers varied somewhat, student motivation, teacher morale, and class size were considered by both groups to be somewhat serious obstacles. The largest number of elementary teachers were concerned about teacher workload, class size, and lack of planning time, while the concerns mentioned by secondary teachers were students' lack of commitment to homework, lack of motivation, and poor attendance. Principals were most concerned about the same three categories as secondary teachers and in addition, students' reading level, though they perceived all categories to be less serious obstacles to instruction than did teachers. These results vary from those sponsored by the National Science Foundation study of math education. Teachers and principals in their sample thought insufficient funds for purchasing equipment and supplies, inadequate facilities, and inadequate articulation of instruction across grade levels were serious problems (Weiss, 1978). The APS sample rated these areas as not very serious obstacles to instruction. Like the NFS sample, APS elementary teachers were also troubled by lack of planning time and secondary teachers in both studies were concerned about student motivation.

Summary & Conclusions

Goals

All of the groups surveyed believed that those individuals closest to the students and with the most specialized knowledge of mathematics, such as classroom teachers and district math specialists, should be the most involved in determining the goals of the APS mathematics program. By the same token, they believed that individuals or groups who were farthest from the classroom or who lacked specialized knowledge of mathematics should not be involved in determining math goals.

The most practical goals were thought to be the most important: teaching basic mathematical knowledge, thinking, and problem solving; preparing students for further education, and for jobs; and enabling students to use math in their daily lives. Less important, but still thought to be somewhat important, were encouraging students to enjoy math for its own sake or in their leisure time.

Content

Four major areas were addressed in the area of the mathematics program content. These were: who should determine math content, the familiarity of various groups with the math program, the adequacy of communication about math, and the appropriateness of the content of the math program at all levels. As with the goals of the math program, survey respondents thought the classroom teacher and the district math specialists should be the most involved in determining the content. In addition, post secondary schools and the math textbook itself were believed to be important determinants of the math curriculum.

All of the groups surveyed reported moderate familiarity with the content of the math program at all levels. However, elementary teachers were less familiar with the content of the math program at the secondary level, than were secondary teachers with the elementary math program. If across-the-board knowledge of the math curriculum is seen as important for adequate student preparation, it may be advantageous to promote teacher knowledge in this area.

In spite of parents' reported familiarity with the math program, a third of the parents felt communication between the school and the home about math was inadequate. Parents at all levels desired information on math content, goals, and expectations so they would have an idea of what their children should master in a year. In addition, while recognizing their responsibility for initiating or improving communication, parents wanted more contact with mathematics teachers in person or in writing. Parents were especially concerned about early notification if their children were experiencing problems in mathematics.

Since personal contact with all parents is probably an impossibility, especially at the high school level, increased written communication seems to be the most appropriate response. Certainly providing parents with information about expectations in mathematics is feasible and may have been

accomplished already at the elementary level through the distribution of competencies booklets to parents in the fall of 1986.

A few areas of the math curriculum need additional emphasis or modification according to school personnel, parents, and professional math organizations. First, the math program at all levels needs to emphasize problem solving skills, the application of mathematics to daily life, mental calculations and estimation, and compilation and interpretation of data. Second, the needs of certain groups of students must be addressed. These are development/remedial students, low achievers, ESL students, and students who are not in the gifted program but need enriched mathematics activities. Third, the high school program needs to be modified in order to expose non-college bound students to a broader range of mathematics and to incorporate consumer mathematics into the preparation of college-bound students. The role of calculus in the high school math program needs further clarification.

Teaching Strategies

Textbook assignments and drill and practice were the most frequently used strategies in teaching mathematics at all grade levels. Elementary math instruction incorporated a broader range of strategies than did secondary math instruction including: textbook assignments, drill and practice, discussion, chalkboard work, small group work, peer tutoring, homework, and student-teacher conferences. At the secondary level, in addition to textbook assignments, homework, lecture, and chalkboard work, teachers also used drill and practice and discussion. Some secondary teachers used peer tutoring and student-teacher conferences. Strategies that were practically never used in mathematics classes at any level were guest speakers, field trips, library work, team teaching, and student projects. In light of the negative shift in attitudes toward mathematics as children progress through school, it is worth considering the impact of teaching strategies and materials. Perhaps more diversity in strategies and materials might improve student interest in mathematics in the upper grades.

Another area that warrants attention is the amount of time spent on mathematics instruction at the elementary level. There was a great deal of variation, from 15 to 60 minutes a day, with an average of 46 minutes. Since Senate Bill 106 madates an hour each day of mathematics instruction in grades one through three, and five hours weekly in grades four and five, instructional time may need to be modified. Clarifying for teachers how to figure instructional time would be helpful since teachers may engage students in mathematics activities in other content areas and may not be including this time in their calculations.

Materials

The textbook is the dominant material used in mathematics instruction at all levels and at the secondary level, practically no other material is used. As with teaching strategies there was a great deal more diversity in the types of materials used in elementary math classes than in secondary classes. In addition to the textbook, elementary teachers frequently used manipulatives, worksheets, and computers. Secondary teachers frequently used other math books, worksheets, and calculators. While calculators were used infrequently at the elementary level, computers and manipulatives were rarely used by secondary teachers. Audiovisual aids were rarely used at any level.

Since the textbook is relied upon so heavily by classroom teachers, it is very important that excellent textbooks are selected, which according to survey respondents and national math organizations should emphasize higher order mathematical skills such as problem solving and applications in addition to computation and basic mathematical knowledge. Also, teachers need to be better informed regarding materials selection procedures, including textbooks.

In addition, an attempt should be made to incorporate more diverse materials into the mathematics classroom by providing teachers with information on the applications of manipulatives, calculators, and computers. Lack of access to these materials may partially explain their limited use, so perhaps the acquisition of more manipulatives and calculators also is necessary.

Teacher Preparation in Mathematics

The results of the questions about teacher preparation for mathematics suggest that math teachers have slightly different needs based on the level they teach. While elementary and secondary teachers both selected math methods and math content as two of the three most important topics of pre-service education, elementary teachers selected sensitivity to student needs while secondary teachers selected classroom management as the third topic. Once teachers are in the classroom, they consider other teachers to be their best source of information. Elementary teachers also found district inservice and the district math specialists to be helpful, while secondary teachers acquired their information more often through publications or meetings of professional math organizations. These results suggest that facilitating interaction among classroom teachers may be a valuable way of improving mathematics instruction. In addition, they suggest that an effort should be made to provide meaningful inservices for all teachers, especially secondary teachers, and to promote the participation of math teachers, especially elementary teachers, in professional math organizations.

Effectiveness of the Math Program

Survey respondents indicated that teacher opinion is the most important in determining the effectiveness of the APS mathematics program. In addition, student achievement on several types of tests was considered important: the APS key competencies, teacher-made tests, and standardized test scores. School personnel and parents rated the APS math program as somewhat effective in achieving all of the goals.

Student achievement results indicate that APS elementary and middle school students score above the national average on standardized tests in mathematics computation and in applications and for the most part have shown steady growth over the last few years. On the Key Competencies, the majority of students master whole numbers, geometry, charts and graphs, fractions, and decimals, while comparatively fewer master measurement and number theory.

Overall, students are fairly positive about mathematics and feel confident of their ability. They believe mathematics is useful outside of school and many of them think they'd enjoy using math in their careers or vocations. Elementary students were the most positive and indicated they enjoy tackling difficult problems and experimenting with mathematics. However, by middle school student attitudes change dramatically. Middle school students were less confident, less interested, and had lost the enthusiasm for problem

solving and experimentation shown by elementary students. The positive attitudes declined slightly between middle school and high school. While some of this may be explained by changes in the students themselves, it is reasonable that at least part of this change in attitude is due to students' experiences in school. This is an area that requires more investigation. Perhaps by altering instructional approaches or by increasing the variety of instructional materials used in the classroom mathematics could be more rewarding, challenging, and enjoyable for students.

Current APS policy insures that staff members, including the project leader, will review the data and findings contained in the report. A plan which includes appropriate steps to address identified program needs will be implemented.

"Darn, I can never guess the ending."

References

Ashlock, R. B. (1976). Error patterns in computation: A semi-programmed approach. Columbus, OH: Charles E. Merrill Publishing Company.

Bell, A. W., Costello, J., & Kuchemann, D. (1983). A review of research in mathematical education. Part A: Research on learning and teaching. Philadelphia, PA: Taylor & Francis Inc.

California State Department of Education. (1982). Handbook for planning an effective mathematics program. Sacramento, CA: Author.

Carpenter, T. P., Coburn, T. G., Reys, R. E., & Wilson, J. W. (October, 1975). Results and implications of the NAEP mathematics assessment: Secondary school. Mathematics Teacher, 68, 453-470.

Carpenter, T. P., Lindquist, M. M., Matthews, W., & Silver, E. A. (December, 1983). Results of the third NAEP mathematics assessment: Secondary school. Mathematics Teacher, 76, 652-659.

Cawelti, G. & Adkisson, J. (1985, April). ASCD study reveals elementary school time allocations for subject areas; other trends noted. ASCD Curriculum Update. Alexandria, VA: Association for Supervision and Curriculum Development.

Chambers, D. L. & Kepner, H. S., Jr. (1985). Applied mathematics: A three-year program for non-college-bound students. In C. R. Hirsch & M. J. Zweng (Eds.), The secondary school mathematics curriculum. Reston, VA: National Council of Teachers of Mathematics.

College Entrance Examination Board. (1985). Academic preparation in mathematics. New York: Author.

Conference Board of the Mathematical Sciences. (1983). The mathematical sciences curriculum K-12: What is still fundamental and what is not. In National Science Board Commission on Precollege Education in Mathematics, Science, and Technology. Washington, DC: National Science Foundation.

Driscoll, M. J. (1980). Research within reach: Elementary school mathematics. Washington, DC: National Institute of Education.

Driscoll, M. J. (1982). Research within reach: Secondary school mathematics. A research-guided response to the concerns of educators. Reston, VA: National Council of Teachers of Mathematics.

Education Commission of the States. (1984). Changes in minimum high school graduation course requirements. ECS Clearinghouse Notes. Denver, CO: Author.

Fey, J. T. (1979, October). Mathematics teaching today. Perspectives from three national surveys. Arithmetic Teacher, 27, 10-14.

Freeman, D. J., Kuhs, T. M., Porter, A. C., Floden, R. E., Schmidt, W. H., Schwille, J. R. (1983). Do textbooks define a national curriculum in elementary school mathematics? The Elementary School Journal, 83 (5), 501-513.

Goodlad, J. (1984). A place called school. New York, NY: McGraw Hill Book Company.

Hurd, P. (1982, May). State of precollege education in mathematics and science. Paper presented at the National Convocation on Precollege Education in Mathematics and Science, Washington, D.C.

Kenelly, J., Henry, P., & Jones, C. O. (1985). The advanced placement program in calculus. In C. R. Hirsch & M. J. Zweng (Eds.), The secondary school mathematics curriculum. Reston, VA: National Council of Teachers of Mathematics.

Lindquist, M. M. (1984). The elementary school mathematics curriculum: Issues for today. The Elementary School Journal, 84 (5), 595-608.

Manhard, W. B., II. (1985). Let's teach mathematics: A case for integrated mathematics programs. In C. R. Hirsch & M. J. Zweng (Eds.), The secondary school mathematics curriculum. Reston, VA: National Council of Teachers of Mathematics.

Naisbitt, J. (1982). Megatrends: Ten new directions transforming our lives. New York, NY: Warner Books.

National Assessment of Educational Progress. (1983). The third national mathematics assessment: Results, trends and issues. (Report No. 13-MA-01). Denver, CO: Education Commission of the States.

National Council of Supervisors of Mathematics. (1978). Position paper on basic mathematical skills. Mathematics Teacher, 71, 147-152.

National Council of Teachers of Mathematics. (1980). An agenda for action: Recommendations for school mathematics of the 1980's. Reston, VA: Author.

National Council of Teachers of Mathematics. (1981). Priorities in school mathematics. Reston, VA: Author.

National Council of Teachers of Mathematics (1985). The impact of computing technology on school mathematics: Report of an NCTM conference. Reston, VA: Author.

Prigge, G. R. (1978, November). The differential effects of the use of manipulative aids on the learning of geometric concepts by elementary school children. Journal for Research in Mathematics Education 9, 361-367.

Raizen, S. A., & Jones, L. V. (Eds.). (1985). Indicators of precollege education in science and mathematics: A preliminary review. Washington, DC: National Academy Press.

Ralston, A. (1985). The really new college mathematics and its impact on the high school curriculum. In C. R. Hirsch & M. J. Zweng (Eds.), The secondary school mathematics curriculum. Reston, VA: National Council of Teachers of Mathematics.

Reeve, W. D. (1955, January). The need for a new national policy and program in secondary mathematics. Mathematics Teacher, 48, 2-9.

Resnick, L. B. (in press). Education and learning to think. Washington, DC: National Research Council.

Reys, R. E., et al. (1980). Hand calculators: What's happening in schools today? Arithmetic Teacher, 27, 38-43.

Sonnabend, T. (1985). Noncareer mathematics: The mathematics we all need. In C. R. Hirsch & M. J. Zweng (Eds.), The secondary school mathematics curriculum. Reston, VA: National Council of Teachers of Mathematics.

Stake, R.E. & Easley, J.A., Jr. (1975). Case studies in science education (NSF SE-78-74). Washington, DC: National Science Foundation.

Suydam, M. N. (1979). The use of calculators in pre-college education: A state-of-the-art review. Columbus, OH: Calculator Information Center.

Suydam, M. N. & Higgins, J. L. (1977). Activity-based learning in elementary school mathematics: Recommendations from research. Reston, VA: National Council of Teachers of Mathematics.

Suydam, M. N. & Osborne, A. (1979). The status of pre-college science, mathematics, and social science education: 1955-1975. Volume II. Mathematics Education. Columbus, OH: The Information Reference Center for Science, Mathematics, and Environmental Education.

Suydam, M. N. & Weaver, E. J. (1975). Research on mathematics learning. In J. N. Payne (Ed.), Mathematics learning in early childhood. 37th NCTM Yearbook. Reston, VA: National Council of Teachers of Mathematics.

Trafton, P. R. (1980). Assessing the mathematics curriculum today. In M. M. Lindquist (Ed.), Selected issues in mathematics education. Berkeley, CA: McCutchan Publishing Corporation.

National Commission on Excellence in Education. (1983). A nation at risk: The imperative for educational reform. Washington, DC: Government Printing Office.

Usiskin, Z. (1983, April). A proposal for reforming the secondary school mathematics curriculum. Paper presented at the 61st Annual Meeting of The National Council of Teachers of Mathematics, Detroit, MI.

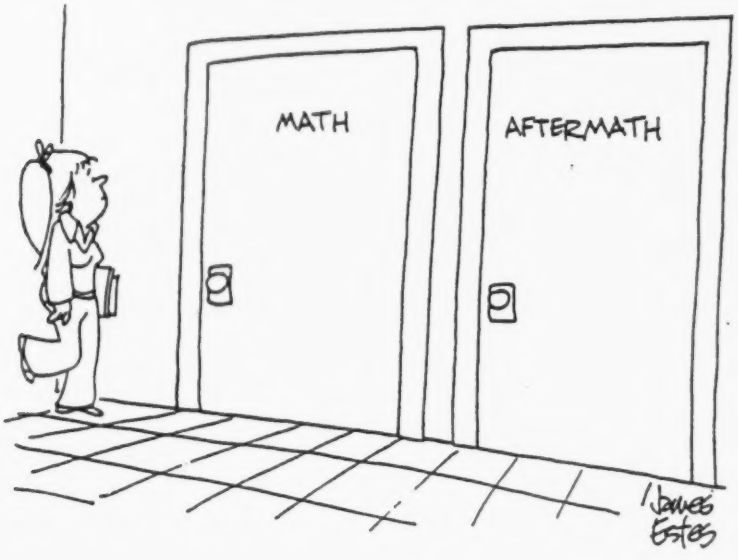
Usiskin, Z. (1985). We need another revolution in secondary school mathematics. In C. R. Hirsh & M. J. Zweng (Eds.), The secondary school mathematics curriculum. Reston, VA: National Council of Teachers of Mathematics.

Walberg, H. J. (1984, May). Improving the productivity of America's schools. Educational Leadership, 23-24.

Weiss, I. R. (1978). Highlights report. 1977 National survey of science, mathematics, and social studies education (Research Triangle Institute 1266/06-02F). Washington, DC: National Science Foundation.

Wisconsin Department of Public Instruction. (1986). A guide to curriculum planning in mathematics. Madison, WI: Author.

Wirtz, et al. (1977). On further examination. Report of the advisory panel on the scholastic aptitude test score decline. New York, NY: College Entrance Examination Board.

Wyatt, T. W., et al. (1979). The status of the hand-held calculator in school. Implications for parents, teachers, and administrators. Phi Delta Kappan, 61, 217-218.

Zarinnia, E. A. & Romberg, T. A. (in press). A new world view and its impact on school mathematics. The monitoring of school mathematics: Background. Madison, WI: Wisconsin Center for Education Research.